Transportation Problem

The transportation problem is a special type of linear programming problem, where the objective is to minimize the cost of distributing a product from a number of sources to a number of destinations.

The transportation problem deals with a special class of linear programming problems in which the objective is to transport  a homogeneous product manufactured at several plants (origins) to a number of different destinations at a minimum total cost. The total supply available at the origin and the total quantity demanded by the destinations are given in the statement of the problem. The cost of shipping a unit of goods from a known origin to a known destination is also given. Our objective is to determine the optimal allocation that results in minimum total shipping cost.

The transportation (or distribution) problem is significant for most commercial organizations that operate several plants and hold inventory in regional warehouses.

Mathematical Representation Of Transportation Problem

A firm has 3 factories - A, E, and K. There are four major warehouses situated at B, C, D, and M. Average daily product at A, E, K is 30, 40, and 50 units respectively. The average daily requirement of this product at B, C, D, and M is 35, 28, 32, 25 units respectively. The transportation cost (in Rs.) per unit of product from each factory to each warehouse is given below:

	Warehouse				
Factory	**B**	**C**	**D**	**M**	**Supply**
A	6	8	8	5	30
E	5	11	9	7	40
K	8	9	7	13	50
Demand	35	28	32	25	

The problem is to determine a routing plan that minimizes total transportation costs.

	B	C	D	M	Supply
A	6 x_{11}	8 x_{12}	8 x_{13}	5 x_{14}	30
E	5 x_{21}	11 x_{22}	9 x_{23}	7 x_{24}	40
K	8 x_{31}	9 x_{32}	7 x_{33}	13 x_{34}	50
Demand	35	28	32	25	

Let x_{ij} = no. of units of a product transported from ith factory(i = 1, 2, 3) to jth warehouse (j = 1, 2, 3, 4).

It should be noted that if in a particular solution the x_{ij} value is missing for a cell, this means that nothing is shipped between factory and warehouse.

The problem can be formulated mathematically in the linear programming form as
Minimize = $6x_{11} + 8x_{12} + 8x_{13} + 5x_{14}$
$+ 5x_{21} + 11x_{22} + 9x_{23} + 7x_{24}$
$+ 8x_{31} + 9x_{32} + 7x_{33} + 13x_{34}$

subject to
Capacity constraints
$x_{11} + x_{12} + x_{13} + x_{14} = 30$
$x_{21} + x_{22} + x_{23} + x_{24} = 40$
$x_{31} + x_{32} + x_{33} + x_{34} = 50$

Requirement constraints
$x_{11} + x_{21} + x_{31} = 35$
$x_{12} + x_{22} + x_{32} = 28$
$x_{13} + x_{23} + x_{33} = 32$
$x_{14} + x_{24} + x_{34} = 25$

$x_{ij} \geq 0$

The above problem has 7 constraints and 12 variables.**Since no. of variables is very high, simplex method is not applicable. Therefore, more efficient methods have been developed to solve transportation problems.**

The general mathematical model may be given as follows

minimize $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}$

subject to

$\sum_{j=1}^{n} x_{ij} \leq S_i$ for i = 1,2, ..., m (supply)

$\sum_{i=1}^{m} x_{ij} \geq D_j$ for j = 1,2, ..., n (demand)

$x_{ij} \geq 0$

For a feasible solution to exist, it is necessary that total capacity equals total requirements.
Total supply = total demand.
Or $\sum a_i = \sum b_j$.

If total supply = total demand then it is a balanced transportation problem.
There will be (m + n – 1) basic independent variables out of (m x n) variables.

? What are the underlying assumptions?

- Only a single type of commodity is being shipped from an origin to a destination.
- Total supply is equal to the total demand.

  $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$
- S_i (supply) and D_j (demand) are all positive integers.

Basic Terminology

In this section, we augment your operations research vocabulary with some new terms.

Origin

It is the location from which shipments are dispatched.

Destination

It is the location to which shipments are transported.

Unit Transportation cost

It is the cost of transporting one unit of the consignment from an origin to a destination.

Perturbation Technique

It is a method used for modifying a degenerate transportation problem, so that the degeneracy can be resolved.

Feasible Solution

A solution that satisfies the row and column sum restrictions and also the non-negativity restrictions is a feasible solution.

Basic Feasible Solution

A feasible solution of (m X n) transportation problem is said to be basic feasible solution, when the total number of allocations is equal to (m + n – 1).

Optimal Solution

A feasible solution is said to be optimal solution when the total transportation cost will be the minimum cost.

In the sections that follow, we will concentrate on algorithms for finding solutions to transportation problems.

Methods for finding an initial basic feasible solution:

- North West Corner Rule
- Matrix Minimum Method
- Vogel Approximation Method

North West Corner Rule

The North West corner rule is a method for computing a basic feasible solution of a transportation problem, where the basic variables are selected from the North – West corner (i.e., top left corner).

The standard North West Corner Rule instructions are paraphrased below:

Steps

1. Select the north west (upper left-hand) corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand, i.e., min(s1, d1).
2. Adjust the supply and demand numbers in the respective rows and columns.
3. If the demand for the first cell is satisfied, then move horizontally to the next cell in the second column.
4. If the supply for the first row is exhausted, then move down to the first cell in the second row.
5. If for any cell, supply equals demand, then the next allocation can be made in cell either in the next row or column.
6. Continue the process until all supply and demand values are exhausted.

This trial routing method is often far from optimal.

Example 1

The Amulya Milk Company has three plants located throughout a state with production capacity 50, 75 and 25 gallons. Each day the firm must furnish its four retail shops R_1, R_2, R_3, & R_4 with at least 20, 20 , 50, and 60 gallons respectively. The transportation costs (in Rs.) are given below.

Plant	**Retail Shop**				**Supply**
	R_1	**R_2**	**R_3**	**R_4**	
P_1	3	5	7	6	50
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	20	20	50	60	

The economic problem is to distribute the available product to different retail shops in such a way so that the total transportation cost is minimum

Solution.

Starting from the North west corner, we allocate min (50, 20) to P_1R_1, i.e., 20 units to cell P_1R_1. The demand for the first column is satisfied. The allocation is shown in the following table.

Table 1

Plant	Retail Shop				Supply
	R_1	R_2	R_3	R_4	
P_1	3 (20)	5	7	6	~~50~~ 30
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	~~20~~	20	50	60	

Now we move horizontally to the second column in the first row and allocate 20 units to cell P_1R_2. The demand for the second column is also satisfied.

Table 2

Plant	Retail Shop				Supply
	R_1	R_2	R_3	R_4	
P_1	3 (20)	5 (20)	7	6	~~50~~ ~~30~~ 10
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	~~20~~	~~20~~	50	60	

Proceeding in this way, we observe that $P_1R_3 = 10$, $P_2R_3 = 40$, $P_2R_4 = 35$, $P_3R_4 = 25$. The resulting feasible solution is shown in the following table.

Final Table

Plant	Retail Shop				Supply
	R_1	R_2	R_3	R_4	
P_1	3 (20)	5 (20)	7 (10)	6	~~50~~
P_2	2	5	8 (40)	2 (35)	~~75~~
P_3	3	6	9	2 (25)	~~25~~
Demand	~~20~~	~~20~~	~~50~~	~~60~~	

Here, number of retail shops(n) = 4, and
Number of plants (m) = 3

Number of basic variables = m + n – 1 = 3 + 4 – 1 = 6.

Initial basic feasible solution

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

20 X 3 + 20 X 5 + 10 X 7 + 40 X 8 + 35 X 2 + 25 X 2 = 670

Example 2

Luminous lamps has three factories - F_1, F_2, and F_3 with production capacity 30, 50, and 20 units per week respectively. These units are to be shipped to four warehouses W_1, W_2, W_3, and W_4 with requirement of 20, 40, 30, and 10 units per week respectively. The transportation costs (in Rs.) per unit between factories and warehouses are given below.

Factory	Warehouse				Supply
	W_1	W_2	W_3	W_4	
F_1	1	2	1	4	30
F_2	3	3	2	1	50

F3	4	2	5	9	20
Demand	20	40	30	10	

Find an initial basic feasible solution of the given transportation problem

Solution.

Starting from the North west corner, we allocate 20 units to F_1W_1. The demand for the first column is completely satisfied.

Table 1

Factory	**Warehouse**				**Supply**
	W1	**W2**	**W3**	**W4**	
F1	1 (20)	2	1	4	~~30~~ 10
F2	3	3	2	1	50
F3	4	2	5	9	20
Demand	~~20~~	40	30	10	

Proceeding in this way, we observe that $F_1W_2 = 10$, $F_2W_2 = 30$, $F_2W_3 = 20$, $F_3W_3 = 10$, $F_3W_4 = 10$. An initial basic feasible solution is exhibited below.

Final Table

Factory	**Warehouse**				**Supply**
	W1	**W2**	**W3**	**W4**	
F1	1 (20)	2 (10)	1	4	~~30~~
F2	3	3 (30)	2 (20)	1	~~50~~
F3	4	2	5 (10)	9 (10)	~~20~~
Demand	~~20~~	~~40~~	~~30~~	~~10~~	

Number of basic variables = $m + n - 1 = 3 + 4 - 1 = 6$.

Initial basic feasible solution

20 X 1 + 10 X 2 + 30 X 3 + 20 X 2 + 10 X 5 + 10 X 9 = 310.

Matrix Minimum Method

Matrix minimum (Least cost) method is a method for computing a basic feasible solution of a transportation problem, where the basic variables are chosen according to the unit cost of transportation. This method is very useful because it reduces the computation and the time required to determine the optimal solution. The following steps summarize the approach.

Steps

1. Identify the box having minimum unit transportation cost (c_{ij}).
2. If the minimum cost is not unique, then you are at liberty to choose any cell.
3. Choose the value of the corresponding x_{ij} as much as possible subject to the capacity and requirement constraints.
4. Repeat steps 1-3 until all restrictions are satisfied.

Example 1

Consider the transportation problem presented in the following table:

Factory	**Retail Shop**				**Supply**
	1	**2**	**3**	**4**	
1	3	5	7	6	50
2	2	5	8	2	75
3	3	6	9	2	25
Demand	20	20	50	60	

Solution.

We observe that c_{21} =2, which is the minimum transportation cost. So x_{21} = 20. The demand for the first column is satisfied. The allocation is shown in the following table.

Table 1

Factory	Retail Shop				Supply
	1	2	3	4	
1	3	5	7	6	50
2	2 (20)	5	8	2	~~75~~ 55
3	3	6	9	2	25
Demand	~~20~~	20	50	60	

Now we observe that c_{24} =2, which is the minimum transportation cost, so x_{24} = 55. The supply for the second row is exhausted.

Table 2

Factory	Retail Shop				Supply
	1	2	3	4	
1	3	5	7	6	50
2	2 (20)	5	8	2 (55)	~~75~~
3	3	6	9	2	25
Demand	~~20~~	20	50	~~60~~ 5	

Proceeding in this way, we observe that x_{34} = 5, x_{12} = 20, x_{13} = 30, x_{33} = 20. The resulting feasible solution is shown in the following table.

Final Table

Factory	Retail Shop				Supply
	1	2	3	4	

1	3	5 (20)	7 (30)	6	~~50~~
2	2 (20)	5	8	2 (55)	~~75~~
3	3	6	9 (20)	2 (5)	~~25~~
Demand	~~20~~	~~20~~	~~50~~	~~60~~	

Number of basic variables = m + n –1 = 3 + 4 – 1 = 6.

Initial basic feasible solution

The total transportation cost associated with this solution is calculated as given below:
20 X 2 + 20 X 5 + 30 X 7 + 55 X 2 + 20 X 9 + 5 X 2 = 650.

Example 2

Consider the transportation problem presented in the following table:

Factory	**Warehouse**			**Supply**
	W_1	**W_2**	**W_3**	
F_1	16	20	12	200
F_2	14	8	18	160
F_3	26	24	16	90
Demand	180	120	150	450

Solution.

We observe that $F_2W_2 = 8$, which is the minimum transportation cost and allocate 120 units to it. The demand for the second column is satisfied.

Table 1

Factory	Warehouse			Supply
	W_1	W_2	W_3	
F_1	16	20	12	200
F_2	14	8 (120)	18	~~160~~ 40
F_3	26	24	16	90
Demand	180	~~120~~	150	450

The resulting feasible solution is shown in the following table.

Final Table

Factory	Warehouse			Supply
	W_1	W_2	W_3	
F_1	16 (50)	20	12 (150)	~~200~~
F_2	14 (40)	8 (120)	18	~~160~~
F_3	26 (90)	24	16	~~90~~
Demand	~~180~~	~~120~~	~~150~~	450

Number of basic variables = m + n –1 = 3 + 3 – 1 = 5.

Initial basic feasible solution

The total transportation cost associated with this solution is calculated as given below:
50 X 16 + 150 X 12 + 40 X 14 + 120 X 8 + 90 X 26 = 6460.

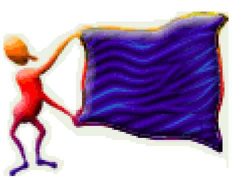
Vogel Approximation Method

The Vogel approximation (Unit penalty) method is an iterative procedure for computing a basic feasible solution of a transportation problem. This method is preferred over the two methods discussed in the previous sections, because the initial basic feasible solution obtained by this method is either optimal or very close to the optimal solution.

Steps

The standard instructions are paraphrased below:

1. Identify the boxes having minimum and next to minimum transportation cost in each row and write the difference (penalty) along the side of the table against the corresponding row.
2. Identify the boxes having minimum and next to minimum transportation cost in each column and write the difference (penalty) against the corresponding column
3. Identify the maximum penalty. If it is along the side of the table, make maximum allotment to the box having minimum cost of transportation in that row. If it is below the table, make maximum allotment to the box having minimum cost of transportation in that column.
4. If the penalties corresponding to two or more rows or columns are equal, you are at liberty to break the tie arbitrarily.
5. Repeat the above steps until all restrictions are satisfied.



This method is a little complex than the previously discussed methods. So go slowly and reread the explanation atleast twice.

Example 1

Consider the transportation problem presented in the following table:

Destination					
Origin	**1**	**2**	**3**	**4**	**Supply**
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Demand	60	40	30	110	240

Solution.

Calculating penalty for table 1

17 - 4 = 13, 9 - 7 = 2, 20 - 15 = 5
24 - 20 = 4, 37 - 22 = 15, 17 - 9 = 8, 7 - 4 = 3

Table 1

Destination						
Origin	**1**	**2**	**3**	**4**	**Supply**	**Penalty**
1	20	40 22	17	4	~~120~~ 80	13
2	24	37	9	7	70	2
3	32	37	20	15	50	5
Demand	60	~~40~~	30	110	240	
Penalty	4	15	8	3		

The highest penalty occurs in the second column. The minimum cij in this column is c_{12} (i.e., 22). So x_{12} = 40 and the second column is **eliminated**. The new reduced matrix is shown below:

Now again calculate the penalty.

Table 2

Origin	**1**	**3**	**4**	**Supply**	**Penalty**

1	20	17	**4** (80)	~~80~~	13
2	24	9	7	70	2
3	32	20	15	50	5
Demand	60	30	110		
Penalty	4	8	3		

The highest penalty occurs in the first row. The minimum c_{ij} in this row is c_{14} (i.e., 4). So x_{14} = 80 and the first row is eliminated. The new reduced matrix is shown below:

Table 3

Origin	1	3	4	Supply	Penalty
2	24	**9** (30)	7	70	2
3	32	20	15	50	5
Demand	60	~~30~~	30		
Penalty	8	11	8		

The highest penalty occurs in the second column. The minimum cij in this column is c_{23} (i.e., 9). So x_{23} = 30 and the second column is eliminated. The reduced matrix is given in the following table.

Table 4

Origin	1	4	Supply	Penalty
2	**24** (10)	**7** (30)	~~40~~	17
3	**32** (50)	15	~~50~~	17
Demand	~~60~~	~~30~~		
Penalty	8	8		

The following table shows the computation of penalty for various rows and columns.

Final table

Destination												
Origin	**1**	**2**	**3**	**4**	**Supply**	**Penalty**						
1	20	22 (40)	17	4 (80)	~~120~~	13	13	-	-	-	-	
2	24 (10)	37	9 (30)	7 (30)	~~70~~	2	2	2	17	24	24	
3	32 (50)	37	20	15	~~50~~	5	5	5	17	32	-	
Demand	~~60~~	~~40~~	~~30~~	~~110~~	240							
Penalty	4	15	8	3								
	4	-	8	3								
	8	-	11	8								
	8	-	-	8								
	8	-	-	-								
	24	-	-	-								

Initial basic feasible solution

22 X 40 + 4 X 80 + 24 X 10 + 9 X 30 + 7 X 30 + 32 X 50 = 3520.

Example 2

Consider the transportation problem presented in the following table:

Destination				
Origin	**1**	**2**	**3**	**Supply**
1	2	7	4	5

2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	34

Solution.

Table 1

Destination					
Origin	**1**	**2**	**3**	**Supply**	**Penalty**
1	2 (5)	7	4	~~5~~	2
2	3	3	1	8	2
3	5	4	7	7	1
4	1	6	2	14	1
Demand	~~7~~ 2	9	18	34	
Penalty	1	1	1		

The highest penalty occurs in the first row. The minimum cij in this row is c_{11} (i.e., 2). Hence, $x_{11} = 5$ and the first row is eliminated.

Now again calculate the penalty. The following table shows the computation of penalty for various rows and columns.

Final table

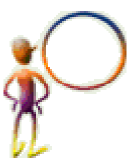
Destination										
Origin	**1**	**2**	**3**	**Supply**	**Penalty**					
1	2 (5)	7	4	~~5~~	2	-	-	-	-	-
2	3	3 (2)	1 (6)	~~8~~	2	2	2	2	3	3
3	5	4 (7)	7	~~7~~	1	1	3	3	4	-

4	1 (2)	6	2 (12)	~~14~~	1	1	4	-	-	-
Demand	~~7~~	~~9~~	~~18~~	34						
Penalty	1	1	1							
	2	1	1							
	-	1	1							
	-	1	6							
	-	1	-							
	-	3	-							

Initial basic feasible solution

5 X 2 + 2 X 3 + 6 X 1 + 7 X 4 + 2 X 1 + 12 X 2 = 76.

Now, you must take a break because you really deserve it. We will see you at the next section when you are ready again.



"Study little but study very thoroughly, because it is thoroughness in work which pays in the long run." - *Anonymous*

Optimality Test

After computing an initial basic feasible solution, we must now proceed to determine whether the solution so obtained is optimal or not. In the next section, we will discuss about the methods used for finding an optimal solution.

Stepping Stone Method

It is a method for finding the optimum solution of a transportation problem.

Steps

1. Determine an initial basic feasible solution using any one of the following:

- North West Corner Rule
- Matrix Minimum Method
- Vogel Approximation Method

2. Make sure that the number of occupied cells is exactly equal to m+n-1, where m is the number of rows and n is the number of columns.

3. Select an unoccupied cell. Beginning at this cell, trace a closed path, starting from the selected unoccupied cell until finally returning to that same unoccupied cell.

The cells at the turning points are called "Stepping Stones" on the path.

4. Assign plus (+) and minus (-) signs alternatively on each corner cell of the closed path just traced, beginning with the plus sign at unoccupied cell to be evaluated.

5. Add the unit transportation costs associated with each of the cell traced in the closed path. This will give net change in terms of cost.

6. Repeat steps 3 to 5 until all unoccupied cells are evaluated.

7. Check the sign of each of the net change in the unit transportation costs. If all the net changes computed are greater than or equal to zero, an optimal solution has been reached. If not, it is possible to improve the current solution and decrease the total transportation cost, so move to step 8..

8. Select the unoccupied cell having the most negative net cost change and determine the maximum number of units that can be assigned to this cell. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell. Add this number to the unoccupied cell and to all other cells on the path marked with a plus sign. Subtract this number from cells on the closed path marked with a minus sign.

For clarity of exposition, consider the following transportation problem.

Example 1

A company has three factories A, B, and C with production capacity 700, 400, and 600 units per week respectively. These units are to be shipped to four depots D, E, F, and G with requirement of 400, 450, 350, and 500 units per week respectively. The transportation costs (in Rs.) per unit between factories and depots are given below:

Depot					
Factory	**D**	**E**	**F**	**G**	**Capacity**
A	4	6	8	6	700
B	3	5	2	5	400
C	3	9	6	5	600
Requirement	400	450	350	500	1700

The decision problem is to minimize the total transportation cost for all factory-depot shipping patterns.

Solution.

An initial basic feasible solution is obtained by Matrix Minimum Method and is shown below in table 1.

Table 1

Depot					
Factory	**D**	**E**	**F**	**G**	**Capacity**
A	4	6 (450)	8	6 (250)	700
B	3 (50)	5	2 (350)	5	400
C	3 (350)	9	6	5 (250)	600
Requirement	400	450	350	500	1700

Here, m + n - 1 = 6. So the solution is not degenerate.

Initial basic feasible solution

6 X 450 + 6 X 250 + 3 X 50 + 2 X 350 + 3 X 350 + 5 X 250 = 7350

Table 2

The cell AD (4) is empty so allocate one unit to it. Now draw a closed path from AD. The result of allocating one unit along with the necessary adjustments in the adjacent cells is indicated in table 2.

	Depot				
Factory	D	E	F	G	Capacity
A	+ 4 (+1)	6 (450)	8	- 6 (249)	700
B	3 (50)	5	2 (350)	5	400
C	- 3 (349)	9	6	+ 5 (251)	600
Requirement	400	450	350	500	1700

Please note that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.

The increase in the transportation cost per unit quantity of reallocation is
+4 – 6 + 5 – 3 = 0.

This indicates that every unit allocated to route AD will neither increase nor decrease the transportation cost. Thus, such a reallocation is unnecessary.

Choose another unoccupied cell. The cell BE is empty so allocate one unit to it.
Now draw a closed path from BE as shown below in table 3.

Table 3

	Depot				
Factory	D	E	F	G	Capacity
A	4	- 6 (449)	8	+ 6 (251)	700
B	- 3 (49)	+ 5 (+1)	2 (350)	5	400
C	+ 3 (351)	9	6	- 5 (249)	600
Requirement	400	450	350	500	1700

The increase in the transportation cost per unit quantity of reallocation is
+5 – 6 + 6 – 5 + 3 – 3 = 0

This indicates that every unit allocated to route BE will neither increase nor decrease the transportation cost. Thus, such a reallocation is unnecessary.

We must evaluate all such unoccupied cells in this manner by finding closed paths and calculating the net cost change as shown below.

Unoccupied cells	Increase in cost per unit of reallocation	Remarks
CE	+9 – 5 + 6 – 6 = 4	Cost Increases
CF	+6 – 3 + 3 – 2 = 4	Cost Increases
AF	+8 – 6 + 5 – 3 + 3 – 2 = 5	Cost Increases
BG	+5 – 5 + 3 – 3 = 0	Neither increase nor decrease

Since all the values of unoccupied cells are greater than or equal to zero, the solution obtained is optimal.

Minimum transportation cost is:
6 X 450 + 6 X 250 + 3 X 50 + 2 X 350 + 3 X 350 + 5 X 250 = Rs. 7350

Confused!!!  **"Furious activity is no substitute for understanding."**

 ***Example 2***

Consider the following transportation problem (cost in rupees)

Distributor				
Factory	**D**	**E**	**F**	**Supply**
A	2	1	5	10

B	7	3	4	25
C	6	5	3	20
Requirement	15	22	18	55

Find out the minimum cost of the given transportation problem.

Solution.

We compute an initial basic feasible solution of the problem by Matrix Minimum Method as shown in table 1.

Table 1

Distributor				
Factory	**D**	**E**	**F**	**Supply**
A	2	1 (10)	5	10
B	7 (13)	3 (12)	4	25
C	6 (2)	5	3 (18)	20
Requirement	15	22	18	55

Here, m + n - 1 = 5. So the solution is not degenerate.

Initial basic feasible solution

1 X 10 + 7 X 13 + 3 X 12 + 6 X 2 + 3 X 18 = 203

The cell AD (2) is empty so allocate one unit to it. Now draw a closed path.

Table 2

Distributor				
Factory	D	E	F	Supply
A	+ 2 (+1)	- 1 (9)	5	10
B	- 7 (12)	+ 3 (13)	4	25
C	6 (2)	5	3 (18)	20
Requirement	15	22	18	55

The increase in the transportation cost per unit quantity of reallocation is:
+ 2 – 1 + 3 – 7 = - 3.

The allocations for other unoccupied cells are following:

Unoccupied cells	Increase in cost per unit of reallocation	Remarks
AF	+5 – 1 + 3 – 7 + 6 – 3 = 3	Cost Increases
CE	+5 – 3 + 7 – 6 = 3	Cost Increases
BF	+4 – 7 + 6 – 3 = 0	Neither increase nor decrease

This indicates that the route through AD would be beneficial to the company. The maximum amount that can be allocated to AD is 10 and this will make the current basic variable corresponding to cell AE non basic.

Table 3 shows the transportation table after reallocation.

Table 3

Distributor				
Factory	D	E	F	Supply
A	2 (10)	1	5	10
B	7 (3)	3 (22)	4	25
C	6 (2)	5	3 (18)	20

Requirement	15	22	18	55

Since the reallocation in any other unoccupied cell cannot decrease the transportation cost, the minimum transportation cost is:
2 X 10 + 7 X 3 + 3 X 22 + 6 X 2 + 3 X 18 = Rs.173

Modified Distribution Method (MODI) or (u - v) method

The modified distribution method, also known as MODI method or (u - v) method provides a minimum cost solution to the transportation problem. In the stepping stone method, we have to draw as many closed paths as equal to the unoccupied cells for their evaluation. To the contrary, in MODI method, only closed path for the unoccupied cell with highest opportunity cost is drawn.

MODI method is an improvement over stepping stone method.

- Steps
- Example

The method, in outline, is :

Steps

1. Determine an initial basic feasible solution using any one of the three methods given below:

- North West Corner Rule
- Matrix Minimum Method
- Vogel Approximation Method

2. Determine the values of dual variables, u_i and v_j, using $u_i + v_j = c_{ij}$

3. Compute the opportunity cost using $c_{ij} – (u_i + v_j)$.

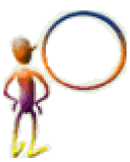
4. Check the sign of each opportunity cost. If the opportunity costs of all the unoccupied cells are either positive or zero, the given solution is the optimal solution. On the other hand, if one or more unoccupied cell has negative opportunity cost, the given solution is not an optimal solution and further savings in transportation cost are possible.

5. Select the unoccupied cell with the smallest negative opportunity cost as the cell to be included in the next solution.

6. Draw a closed path or loop for the unoccupied cell selected in the previous step. Please note that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.

7. Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated.

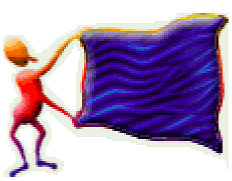
8. Determine the maximum number of units that should be shipped to this unoccupied cell. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell. Now, **add** this quantity to all the cells on the corner points of the closed path marked with plus signs, and **subtract** it from those cells marked with minus signs. In this way, an unoccupied cell becomes an occupied cell.



"A man has a burger and you give him one burger more, that's addition." -*Vinay Chhabra & Manish Dewan*

9. Repeat the whole procedure until an optimal solution is obtained.

Modified Distribution Method - Example



This example is the largest and the most involved you have read so far. So you must read the steps and the explanation mindfully.

Example

Consider the transportation problem presented in the following table.

Distribution centre						
		D1	**D2**	**D3**	**D4**	**Supply**

Plant	**P1**	19	30	50	12	7
	P2	70	30	40	60	10
	P3	40	10	60	20	18
Requirement		5	8	7	15	

Determine the optimal solution of the above problem.

Solution.

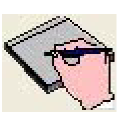
An initial basic feasible solution is obtained by Matrix Minimum Method and is shown in table 1.

Table 1

Distribution centre						
		D1	**D2**	**D3**	**D4**	**Supply**
Plant	**P1**	19	30	50	12 (7)	7
	P2	70 (3)	30	40 (7)	60	10
	P3	40 (2)	10 (8)	60	20 (8)	18
Requirement		5	8	7	15	

Initial basic feasible solution

12 X 7 + 70 X 3 + 40 X 7 + 40 X 2 + 10 X 8 + 20 X 8 = Rs. 894.

***Calculating u_i and v_j using $u_i + v_j = c_{ij}$***

Substituting $u_1 = 0$, we get

$u_1 + v_4 = c_{14} \Rightarrow 0 + v_4 = 12$ or $v_4 = 12$
$u_3 + v_4 = c_{34} \Rightarrow u_3 + 12 = 20$ or $u_3 = 8$
$u_3 + v_2 = c_{32} \Rightarrow 8 + v_2 = 10$ or $v_2 = 2$

$u_3 + v_1 = c_{31} \Rightarrow 8 + v_1 = 40$ or $v_1 = 32$
$u_2 + v_1 = c_{21} \Rightarrow u_2 + 32 = 70$ or $u_2 = 38$
$u_2 + v_3 = c_{23} \Rightarrow 38 + v_3 = 40$ or $v_3 = 2$

Table 2

Distribution centre							
		D1	**D2**	**D3**	**D4**	**Supply**	**u_i**
	P1	19	30	50	12 (7)	7	0
Plant	**P2**	70 (3)	30	40 (7)	60	10	38
	P3	40 (2)	10 (8)	60	20 (8)	18	8
Requirement		5	8	7	15		
v_j		32	2	2	12		

Calculating opportunity cost using $c_{ij} - (u_i + v_j)$

Unoccupied cells	**Opportunity cost**
(P_1, D_1)	$c_{11} - (u_1 + v_1) = 19 - (0 + 32) = -13$
(P_1, D_2)	$c_{12} - (u_1 + v_2) = 30 - (0 + 2) = 28$
(P_1, D_3)	$c_{13} - (u_1 + v_3) = 50 - (0 + 2) = 48$
(P_2, D_2)	$c_{22} - (u_2 + v_2) = 30 - (38 + 2) = -10$
(P_2, D_4)	$c_{14} - (u_2 + v_4) = 60 - (38 + 12) = 10$
(P_3, D_3)	$c_{33} - (u_3 + v_3) = 60 - (8 + 2) = 50$

Table 3

Distribution centre							
		D1	**D2**	**D3**	**D4**	**Supply**	**u_i**

Plant	P1	-13 19	28 30	48 50	12 (7)	7	0
	P2	70 (3)	-10 30	40 (7)	10 60	10	38
	P3	40 (2)	10 (8)	50 60	20 (8)	18	8
Requirement		5	8	7	15		
v_j		32	2	2	12		

Now choose the smallest (most) negative value from opportunity cost (i.e., –13) and draw a closed path from P1D1. The following table shows the closed path.

Table 4

Distribution centre							
		D1	D2	D3	D4	Supply	u_i
Plant	P1	-13 19 +	28 30	48 50	- 12 (7)	7	0
	P2	70 (3)	-10 30	40 (7)	10 60	10	38
	P3	40 (2) -	10 (8)	50 60	+ 20 (8)	18	8
Requirement		5	8	7	15		
v_j		32	2	2	12		

Choose the smallest value with a negative position on the closed path(i.e., 2), it indicates the number of units that can be shipped to the entering cell. Now add this quantity to all the cells on the corner points of the closed path marked with plus signs and subtract it from those cells marked with minus signs. In this way, an unoccupied cell becomes an occupied cell.

Now again calculate the values for u_i & v_j and opportunity cost. The resulting matrix is shown below.

Table 5

Distribution centre							
		D1	**D2**	**D3**	**D4**	**Supply**	u_i
Plant	**P1**	19 (2)	28 30	61 50	12 (5)	7	0
	P2	70 (3)	-23 30	40 (7)	-3 60	10	51

	P3	13 40	10 (8)	63 60	20 (10)	18	8
Requirement		5	8	7	15		
v_j		19	2	–11	12		

Choose the smallest (most) negative value from opportunity cost (i.e., –23). Now draw a closed path from P2D2 .

Table 6

Distribution centre							
		D1	D2	D3	D4	Supply	u_i
Plant	P1	+ 19 (2)	28 30	61 50	− 12 (5)	7	0
	P2	− 70 (3)	-23 30 +	40 (7)	-3 60	10	51
	P3	13 40	− 10 (8)	63 60	+ 20 (10)	18	8
Requirement		5	8	7	15		
v_j		19	2	–11	12		

Now again calculate the values for u_i & v_j and opportunity cost

Don't panic. The following table is the last table.

Table 7

Distribution centre							
		D1	**D2**	**D3**	**D4**	**Supply**	**u_i**
Plant	**P1**	19 (5)	28 30	38 50	12 (2)	7	0

	P2	23 70	30 (3)	40 (7)	20 60	10	28
	P3	13 40	10 (5)	40 60	20 (13)	18	8
Requirement		5	8	7	15		
v_j		19	2	12	12		

Since all the current opportunity costs are non–negative, this is the optimal solution. The minimum transportation cost is: 19 X 5 + 12 X 2 + 30 X 3 + 40 X 7 + 10 X 5 + 20 X 13 = Rs. 799

Degeneracy

If the basic feasible solution of a transportation problem with m origins and n destinations has fewer than m + n – 1 positive x_{ij} (occupied cells), the problem is said to be a degenerate transportation problem.

Degeneracy can occur at two stages:

1. At the initial solution
2. During the testing of the optimal solution

To resolve degeneracy, we make use of an artificial quantity (*d*). The quantity *d* is assigned to that unoccupied cell, which has the minimum transportation cost.

For calculation purposes, the value of *d* is assumed to be zero.

The use of *d* is illustrated in the following example.

Example

Factory	**Dealer**				**Supply**
	1	**2**	**3**	**4**	
A	2	2	2	4	1000
B	4	6	4	3	700

C	3	2	1	0	900
Requirement	900	800	500	400	

Solution.

An initial basic feasible solution is obtained by Matrix Minimum Method.

Table 1

Factory	**Dealer**				**Supply**
	1	**2**	**3**	**4**	
A	2 (900)	2 (100)	2	4	1000
B	4	6 (700)	4	3	700
C	3	2	1 (500)	0 (400)	900
Requirement	900	800	500	400	

Number of basic variables = m + n – 1 = 3 + 4 – 1 = 6
Since number of basic variables is less than 6, therefore, it is a degenerate transportation problem.

To resolve degeneracy, we make use of an artificial quantity(d). The quantity d is assigned to that unoccupied cell, which has the minimum transportation cost.

The quantity d is so small that it does not affect the supply and demand constraints.

In the above table, there is a tie in selecting the smallest unoccupied cell. In this situation, you can choose any cell arbitrarily. We select the cell C2 as shown in the following table.

Table 2

Factory	**Dealer**				**Supply**
	1	**2**	**3**	**4**	

A	2 (900)	2 (100)	2	4	1000
B	4	6 (700)	4	3	700
C	3	2 (d)	1 (500)	0 (400)	900 + d
Requirement	900	800 + d	500	400	2600 + d

Now, we use the stepping stone method to find an optimal solution.

Calculating opportunity cost

Unoccupied cells	Increase in cost per unit of reallocation	Remarks
A3	+2 – 2 + 2 – 1 = 1	Cost Increases
A4	+4 – 2 + 2 – 0 = 4	Cost Increases
B1	+4 – 6 + 2 – 2 = –2	Cost Decreases
B3	+4 – 6 + 2 – 1 = –1	Cost Decreases
B4	+3 – 6 + 2 – 0 = –1	Cost Decreases
C1	+3 – 2 + 2 – 2 = 1	Cost Increases

The cell B1 is having the maximum improvement potential, which is equal to -2. The maximum amount that can be allocated to B1 is 700 and this will make the current basic variable corresponding to cell B2 non basic. The improved solution is shown in the following table.

Table 3

Factory	Dealer				Supply
	1	2	3	4	
A	2 (200)	2 (800)	2	4	1000
B	4 (700)	6	4	3	700
C	3	2 (d)	1 (500)	0 (400)	900

Requirement	900	800	500	400	2600

The optimal solution is
2 X 200 + 2 X 800 + 4 X 700 + 2 X d + 1 X 500 + 0 X 400 = 5300 + 2d.
Notice that *d* is a very small quantity so it can be neglected in the optimal solution. Thus, the net transportation cost is Rs. 5300

Unbalanced Transportation Problem

So far we have assumed that the total supply at the origins is equal to the total requirement at the destinations.
Specifically,

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$$

But in certain situations, the total supply is not equal to the total demand. Thus, the transportation problem with unequal supply and demand is said to be unbalanced transportation problem.

? How we can solve an unbalanced transportation problem?

If the total supply is more than the total demand, we introduce an additional column, which will indicate the surplus supply with transportation cost zero. Similarly, if the total demand is more than the total supply, an additional row is introduced in the table, which represents unsatisfied demand with transportation cost zero. The balancing of an unbalanced transportation problem is illustrated in the following example.

Example

Plant	**Warehouse**			**Supply**
	W1	**W2**	**W3**	
A	28	17	26	500
B	19	12	16	300
Demand	250	250	500	

Solution:

The total demand is 1000, whereas the total supply is 800.

$\sum_{i=1}^{m} S_i < \sum_{j=1}^{n} D_j$

Total supply < total demand.

To solve the problem, we introduce an additional row with transportation cost zero indicating the unsatisfied demand.

Plant	Warehouse			Supply
	W1	W2	W3	
A	28	17	26	500
B	19	12	16	300
Unsatisfied demand	0	0	0	200
Demand	250	250	500	1000

Using matrix minimum method, we get the following allocations.

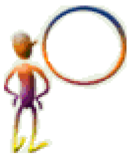
Plant	Warehouse			Supply
	W1	W2	W3	
A	(50) 28	17	(450) 26	~~500~~
B	19	(250) 12	(50) 16	~~300~~
Unsatisfied demand	(200) 0	0	0	~~200~~
Demand	~~250~~	~~250~~	~~500~~	1000

Initial basic feasible solution

50 X 28 + 450 X 26 + 250 X 12 + 50 X 16 + 200 X 0 = 16900.

Maximization In A Transportation Problem

There are certain types of transportation problems where the objective function is to be maximized instead of being minimized. These problems can be solved by converting the maximization problem into a minimization problem.



"Profit maximization is the single universal objective for most commercial organizations."
-Vinay Chhabra & Manish Dewan

Example

Surya Roshni Ltd. has three factories - X, Y, and Z. It supplies goods to four dealers spread all over the country. The production capacities of these factories are 200, 500 and 300 per month respectively.

Factory	**Dealer**				**Capacity**
	A	**B**	**C**	**D**	
X	12	18	6	25	200
Y	8	7	10	18	500
Z	14	3	11	20	300
Demand	180	320	100	400	

Determine a suitable allocation to maximize the total net return.

Solution.

Maximization transportation problem can be converted into minimization transportation problem by subtracting each transportation cost from maximum transportation cost.

Here, the maximum transportation cost is 25. So subtract each value from 25. The revised transportation problem is shown below.

Table 1

Factory	**Dealer**	**Capacity**

	A	B	C	D	
X	13	7	19	0	200
Y	17	18	15	7	500
Z	11	22	14	5	300
Demand	180	320	100	400	

An initial basic feasible solution is obtained by matrix-minimum method and is shown in the final table.

Final table

Factory	**Dealer**				**Capacity**
	A	**B**	**C**	**D**	
X	13	7	19	0 (200)	~~200~~
Y	17 (80)	18 (320)	15 (100)	7	~~500~~
Z	11 (100)	22	14	5 (200)	~~300~~
Demand	~~180~~	~~320~~	~~100~~	~~400~~	

The maximum net return is

25 X 200 + 8 X 80 + 7 X 320 + 10 X 100 + 14 X 100 + 20 X 200 = 14280.

Prohibited Routes

Sometimes there may be situations, where it is not possible to use certain routes in a transportation problem. For example,  road construction, bad road conditions, strike, unexpected floods, local traffic rules, etc. We can handle such type of problems in different ways:

- A very large cost represented by M or $\propto$ is assigned to each of such routes, which are not available.
- To block the allocation to a cell with a prohibited route, we can cross out that cell.

The problem can then be solved in its usual way.

Example

Consider the following transportation problem.

Factory	Warehouse			Supply
	W_1	W_2	W_3	
F_1	16	∝	12	200
F_2	14	8	18	160
F_3	26	∝	16	90
Demand	180	120	150	450

Solution.

An initial solution is obtained by the matrix minimum method and is shown in the final table.

Final Table

Factory	Warehouse			Supply
	W_1	W_2	W_3	
F_1	16 (50)	∝	12 (150)	~~200~~
F_2	14 (40)	8 (120)	18	~~160~~
F_3	26 (90)	∝	16	~~90~~
Demand	~~180~~	~~120~~	~~150~~	450

Initial basic feasible solution

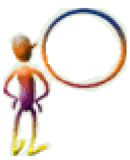
16 X 50 + 12 X 150 + 14 X 40 + 8 X 120 + 26 X 90 = 6460.

The minimum transportation cost is Rs. 6460.

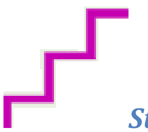
Time Minimizing Problem

Succinctly, it is a transportation problem in which the objective is to minimize the time. This problem is same as the transportation problem of minimizing the cost, expect that the unit transportation cost is replaced by the time t_{ij}.

In a cost minimization problem, the cost of transportation depends on the quantity shipped. To the contrary, in a time minimization problem, the time involved is independent of the amount of commodity shipped.



"One thing you can't recycle is wasted time."
-Anon.



Steps

1. Determine an initial basic feasible solution using any one of the following:

- North West Corner Rule
- Matrix Minimum Method
- Vogel Approximation Method

2. Find T_k for this feasible plan and cross out all the unoccupied cells for which $t_{ij} \geq T_k$.

3. Trace a closed path for the occupied cells corresponding to T_k. If no such closed path can be formed, the solution obtained is optimum otherwise, go to step 2.

Example 1

The following matrix gives data concerning the transportation times t_{ij}

Destination							
Origin	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**	**Supply**
O1	25	30	20	40	45	37	37

O2	30	25	20	30	40	20	22
O3	40	20	40	35	45	22	32
O4	25	24	50	27	30	25	14
Demand	15	20	15	25	20	10	

Solution.

We compute an initial basic feasible solution by north west corner rule which is shown in table 1.

Table 1

Destination							
Origin	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**	**Supply**
O1	25 (15)	30 (20)	20 (2)	40	45	37	37
O2	30	25	20 (13)	30 (9)	40	20	22
O3	40	20	40	35 (16)	45 (16)	22	32
O4	25	24	50	27	30 (4)	25 (10)	14
Demand	15	20	15	25	20	10	

Here, t_{11} = 25, t_{12} = 30, t_{13} = 20, t_{23} = 20, t_{24} = 30, t_{34}= 35, t_{35} = 45, t_{45} =30, t_{46} = 25

Choose maximum from t_{ij}, i.e., T_1 = 45. Now, cross out all the unoccupied cells that are ≥ T1. The unoccupied cell (O3D6) enters into the basis as shown in table 2.

Table 2

Destination							
Origin	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**	**Supply**
O1	25 (15)	30 (20)	20 (2)	40	~~45~~	37	37
O2	30	25	20 (13)	30 (9)	40	20	22
O3	40	20	40	35 (16)	− 45 (16)	+ 22	32
O4	25	24	~~50~~	27	+ 30 (4)	− 25 (10)	14
Demand	15	20	15	25	20	10	

Choose the smallest value with a negative position on the closed path, i.e., 10. Clearly only 10 units can be shifted to the entering cell. The next feasible plan is shown in the following table.

Table 3

Destination							
Origin	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**	**Supply**
O1	25 (15)	30 (20)	20 (2)	40	~~45~~	37	37
O2	30	25	20 (13)	30 (9)	40	20	22
O3	40	20	40	35 (16)	45 (6)	22 (10)	32
O4	25	24	~~50~~	27	30 (14)	25	14
Demand	15	20	15	25	20	10	

Here, T_2 = Max(25, 30, 20, 20, 20, 35, 45, 22, 30) = 45. Now, cross out all the unoccupied cells that are $\geq T_2$.

Table 4

	Destination						
Origin	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**	**Supply**
O1	25 (15)	30 (20)	20 (2)	40	~~45~~	37	37
O2	30	25	20 (13)	- 30 (9)	+ 40	20	22
O3	40	20	40	+ 35 (16)	- 45 (6)	22 (10)	32
O4	25	24	~~50~~	27	30 (14)	25	14
Demand	15	20	15	25	20	10	

By following the same procedure as explained above, we get the following revised matrix.

Table 6

	Destination						
Origin	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**	**Supply**
O1	25 (15)	30 (20)	20 (2)	~~40~~	~~45~~	37	37
O2	30	25	20 (13)	30 (3)	40 (6)	20	22
O3	~~40~~	20	~~40~~	35 (22)	~~45~~	22 (10)	32
O4	25	24	~~50~~	27	30 (14)	25	14
Demand	15	20	15	25	20	10	

T_3 = Max(25, 30, 20, 20, 30, 40, 35, 22, 30) = 40. Now, cross out all the unoccupied cells that are $\geq T_3$.

Now we cannot form any other closed loop with T_3.
Hence, the solution obtained at this stage is optimal.
Thus, all the shipments can be made within 40 units.

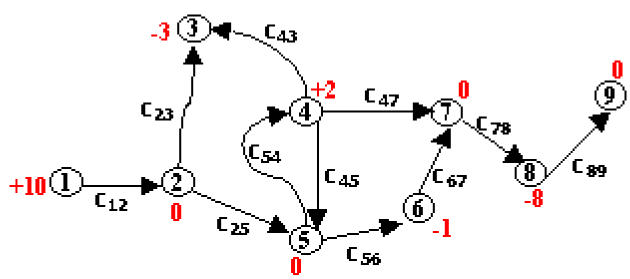
Transshipment Model

In a transportation problem, consignments are always transported from an origin to a destination. But, there could be several situations where it might be economical to transport items via one or more intermediate centres (or stages). In a transshipment problem, the available commodity is not sent directly from sources to destinations, i.e., it passes through one or more intermediate points before reaching the actual destination. For instance, a company may have regional warehouses that distribute the products to smaller district warehouses, which in turn ship to the retail stores. Succinctly, the transshipment model is an extension of the classical transportation model where an item available at point i is shipped to demand point j through one or more intermediate points.

The transshipment model helps the management of a company in deciding the optimal number and location of its warehouses.

Example

A company has nine large stores located in several states. The sales department is interested in reducing the price of a certain product in order to dispose all the stock now in hand. But, before that the management wants to reposition its stock among the nine stores according to its sales expectations at each location.



The above figure shows the numbered nodes (9 stores). A positive value next to a store represents the amount of inventory to be redistributed to the rest of the system. A negative value represents the shortage of stock. Thus, stores 1 and 4 have excess stock of 10 & 2 items respectively. Stores 3, 6 & 8 need 3, 1, and 8 more items respectively. The inventory positions of stores 2, 5 & 7 are to remain unchanged.

An item may be shipped through stores 2, 4, 5, 6, 7 & 8. These locations are known as transshipment points. Each remaining store is a **source** if it has excess stock, and a **sink** if it needs stock. In the above figure, store 1 is a source and store 3 is a sink.

The value c_{ij} is the cost of transporting items. To transport an item from store 1 to store 3, the total shipping cost is

$\mathbf{c_{12} + c_{23}}$

In the following example, you will learn how to convert a transshipment problem to a standard transportation problem.

Example

Consider a transportation problem where the origins are plants and destinations are depots. The unit transportation costs, capacity at the plants, and the requirements at the depots are indicated below:

Table 1

Plant	Depot			
	X	Y	Z	
A	1	3	15	150
B	3	5	25	300
	150	150	150	450

When each plant is also considered a destination and each depot is also considered an origin, there are altogether five origins and five destinations. Some additional cost data are also necessary. These are presented in the following Tables.

Table 2

Unit Transportation Cost from Plant to Plant		
From Plant	To	
	Plant A	Plant B
A	0	65
B	1	0

Table 3

Unit Transportation Cost from Depot to Depot			
From Depot	To		
	Depot X	Depot Y	Depot Z
X	0	23	1

Y	1	0	3
Z	65	3	0

Table 4

Unit Transportation Cost from Depot to Plant		
Depot	**Plant**	
	A	**B**
X	3	15
Y	25	3
Z	45	55

Solution.

From Table 1, Table 2, Table 3 and Table 4 we obtain the transportation formulation of the transshipment problem.

Table 5

Transshipment Table						
	A	**B**	**X**	**Y**	**Z**	**Capacity**
A	0	65	1	3	15	150 + 450 = 600
B	1	0	3	5	25	300 + 450 = 750
X	3	15	0	23	1	450
Y	25	3	1	0	3	450
Z	45	55	65	3	0	450
Requirement	450	450	150 + 450 =600	150 + 450 =600	150 + 450 =600	2700

The transportation model is extended and now it includes five supply points & demand points. To have a supply and demand from all the points, a fictitious supply and demand quantity (**buffer stock**) of 450 is added to both supply and demand of all the points. An initial basic feasible solution is obtained by the **Vogel's Approximation method** and is shown in the final table.

Final Table

Transshipment Table						
	A	**B**	**X**	**Y**	**Z**	**Capacity**
A	(150) 0	65	(300) 1	(150) 3	15	600
B	(300) 1	(450) 0	3	5	25	750
X	3	15	(300) 0	23	(150) 1	450
Y	25	3	1	(450) 0	3	450
Z	45	55	65	3	(450) 0	450
Requirement	450	450	600	600	600	2700

The total transhipment cost is:
0 X 150 + 1 X 300 + 3 X 150 + 1 X 300 + 0 X 450 + 0 X 300 + 1 X 150 + 0 X 450 + 0 X 450 = 1200