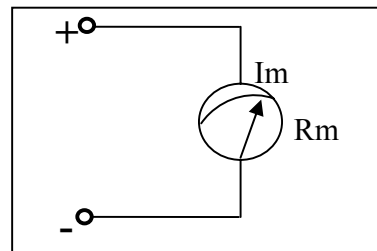


1- D.c Ammeter:

An Ammeter is always connected in series with a circuit branch and measures the current flowing in it. Most d.c ammeters employ a d'Arsonval movement, an ideal ammeter would be capable of performing the measurement without changing or distributing the current in the branch but real ammeters would possess some internal resistance.



Extension of Ammeter Range:

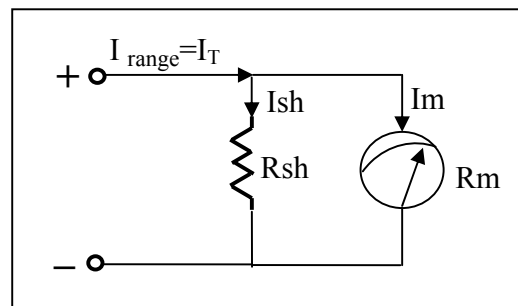
Since the coil winding in PMMC meter is *small and light*, they can carry only small currents (μA - 1mA). Measurement of large current requires **a shunt external resistor** to connect with the meter movement, so only a fraction of the total current will pass through the meter.

$$V_m = V_{sh}$$

$$I_m R_m = I_{sh} R_{sh}$$

$$I_{sh} = I_T - I_m$$

$$R_{sh} = \frac{I_m R_m}{I_T - I_m}$$



Example:

If PMMC meter have internal resistance of 10Ω and full scale range of 1mA . Assume we wish to increase the meter range to 1A .

Sol.

So we must connect shunt resistance with the PMMC meter of

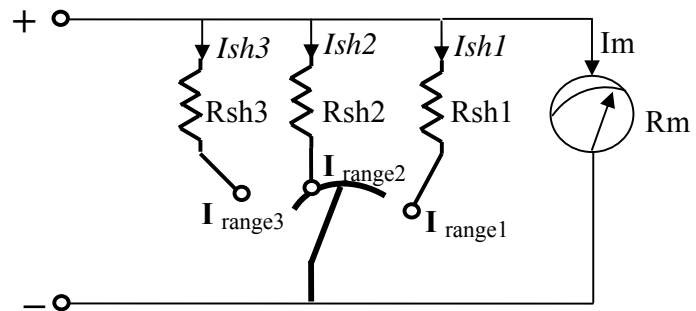
$$R_{sh} = \frac{I_m R_m}{I_T - I_m}$$

$$R_{sh} = \frac{1 \times 10^{-3} \cdot 10}{1 - 1 \times 10^{-3}} = 0.01001\Omega$$

a) Direct D.c Ammeter Method (Ayrton Shunt):

The current range of d.c ammeter can be further extended by a number of shunts selected by a range switch; such ammeter is called a multirange ammeter.

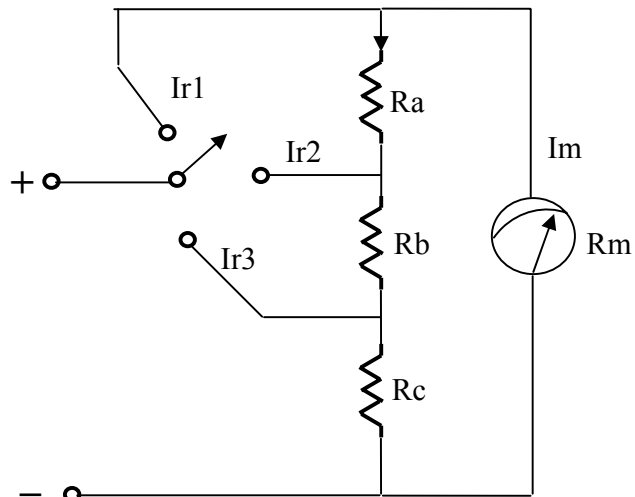
$$R_{sh*} = \frac{I_m R_m}{I_{r*} - I_m}$$



b) Indirect D.C Ammeter Method:

$$\frac{I_{r*}}{I_m} = \frac{R_m + R}{r*}$$

Where $R = R_a + R_b + R_c$
 And $r =$ parallel resistors
 branch with the meter



Example (1):

Design a multirange ammeter by using **direct method** to give the following ranges 10mA, 100mA, 1A, 10A, and 100A. If d'Arsonval meter have internal resistance of 10Ω and full scale current of 1mA.

Sol:

$R_m = 10\Omega \quad I_m = 1\text{mA}$

$$R_{sh*} = \frac{I_m R_m}{I_* - I_m}$$

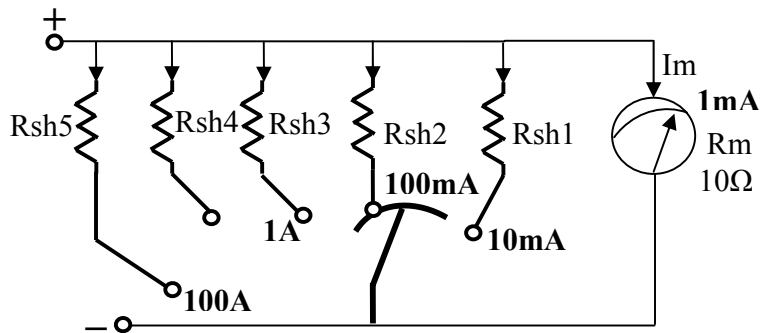
$$R_{sh1} = \frac{1 \times 10^{-3} \cdot 10}{(10 - 1) \times 10^{-3}} = 1.11\Omega$$

$$R_{sh2} = \frac{1 \times 10^{-3} \cdot 10}{(100 - 10) \times 10^{-3}} = 0.101\Omega$$

$$R_{sh3} = \frac{1 \times 10^{-3} \cdot 10}{1 - 10 \times 10^{-3}} = 0.0101\Omega$$

$$R_{sh4} = \frac{1 \times 10^{-3} \cdot 10}{10 - 1 \times 10^{-3}} = 0.0011\Omega$$

$$R_{sh5} = \frac{1 \times 10^{-3} \cdot 10}{100 - 1 \times 10^{-3}} = 0.00011\Omega$$



Example (2):

Design an Ayrton shunt by **indirect method** to provide an ammeter with current ranges 1A, 5A, and 10A, if PMMC meter have internal resistance of 50Ω and full scale current of 1mA.

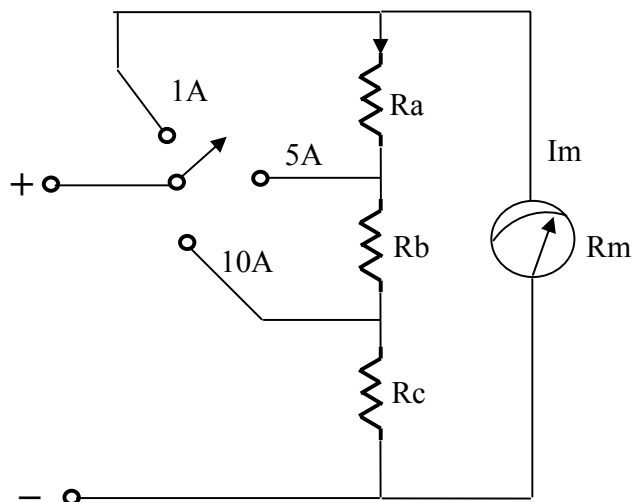
Sol:

$R_m = 50\Omega \quad I_{FSD} = I_m = 1\text{mA}$

$$\frac{I_*}{I_m} = \frac{R_m + R}{r_*}$$

Where $R = R_a + R_b + R_c$

And $r =$ parallel resistors branch with the meter



1- For 1A Range:

$$\frac{I_1}{I_m} = \frac{R_m + R}{R}$$

$$\frac{1A}{1mA} = \frac{50 + R}{R} \quad R=0.05005\Omega$$

2- For 5A Range:

$$\frac{I2}{Im} = \frac{Rm + R}{Rb + Rc} \quad r = Rb + Rc$$

$$\frac{5A}{1mA} = \frac{50 + 0.05005}{Rb + Rc} \quad Rb + Rc = 0.01001\Omega$$

$$Ra = R - (Rb + Rc) \quad Ra = 0.05 - 0.01001 = 0.04004 \Omega$$

3- For 10A Range:

$$\frac{I3}{Im} = \frac{Rm + R}{Rc} \quad r = Rc$$

$$\frac{10A}{1mA} = \frac{50 + 0.05005}{Rc} \quad Rc = 5.005 \times 10^{-3} \Omega$$

$$Rb = 0.01001 - 5.005 \times 10^{-3} = 5.005 \times 10^{-3} \Omega$$

2- D.C Voltmeter:

A voltmeter is always connect in parallel with the element being measured, and measures the voltage between the points across which its' connected. Most d.c voltmeter employ PMMC meter with series resistor as shown. The series resistance should be much larger than the impedance of the circuit being measured, and they are usually much larger than R_m .

$$Rs = RT - Rm$$

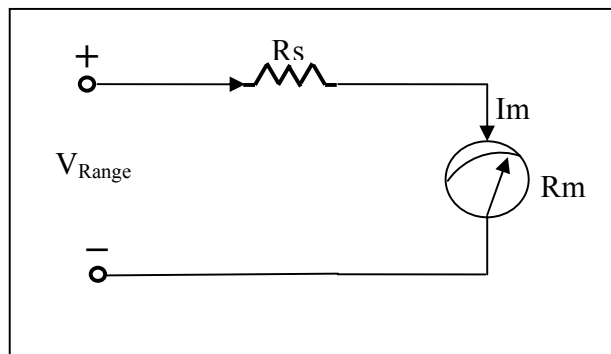
$$Rs = \frac{V_{range}}{Im} - Rm$$

$$Im = I_{FSD}$$

The ohm/volt sensitivity of a voltmeter

Is given by:

$$S_v = \frac{Rm}{V_{FSD}} = \frac{1}{I_{FSD}} = \frac{\Omega}{V} \text{ rating}$$



$$S_{Range} = \frac{Rm + Rs}{V_{Range}} = \frac{1}{I_{Range}} = \frac{\Omega}{V}$$

So the internal resistance of voltmeter or the input resistance of voltmeter is

$$Rv = V_{FSD} \times \text{sensitivity}$$

Example:

We have a micro ammeter and we wish to adapted it so as to measure 1 volt full scale, the meter has internal resistance of 100Ω and I_{FSD} of $100\mu A$.

Sol.:

$$R_s = \frac{V}{I_m} - R_m$$

$$R_s = \frac{1}{0.0001} - 100 = 9900\Omega = 9.9K\Omega$$

So we connect with PMMC meter a series resistance of 9.9KΩ to convert it to voltmeter

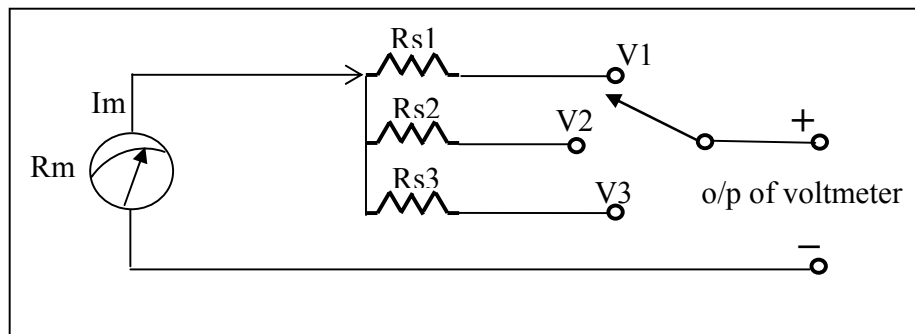
Extension of Voltmeter Range:

Voltage range of d.c voltmeter can be further extended by a number of series resistance selected by a range switch; such a voltmeter is called multirange voltmeter.

a) Direct D.c Voltmeter Method:

In this method each series resistance of multirange voltmeter is connected in direct with PMMC meter to give the desired range.

$$R_{s*} = \frac{V_*}{I_m} - R_m$$



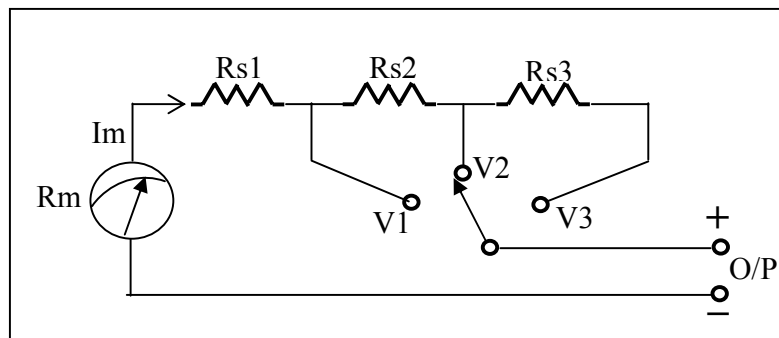
b) Indirect D.c Voltmeter Method:

In this method one or more series resistances of multirange voltmeter is connected with PMMC meter to give the desired range.

$$R_{s1} = \frac{V1}{I_m} - R_m$$

$$R_{s2} = \frac{V2 - V1}{I_m}$$

$$R_{s3} = \frac{V3 - V2}{I_m}$$



Example (1):

A basic d'Arsonval movement with internal resistance of 100Ω and half scale current deflection of 0.5 mA is to be converted by indirect method into a multirange d.c voltmeter with voltages ranges of 10V, 50V, 250V, and 500V.

Sol.:

$$I_{FSD} = I_{HSD} \times 2$$

$$I_{FSD} = 0.5mA \times 2 = 1mA$$

$$R_{s1} = \frac{V1}{I_m} - R_m$$

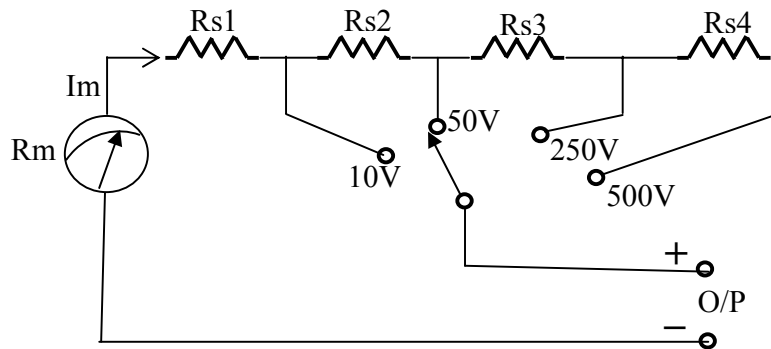
$$R_{s1} = \frac{10}{1mA} - 100 = 9.9K\Omega$$

$$R_{s2} = \frac{V_2 - V_1}{I_m}$$

$$R_{s2} = \frac{50 - 10}{1 \times 10^{-3}} = 40 K\Omega$$

$$R_{s3} = \frac{250 - 50}{1 \times 10^{-3}} = 200 K\Omega$$

$$R_{s4} = \frac{500 - 250}{1 \times 10^{-3}} = 250 K\Omega$$



Example (2):

Design d.c voltmeter by using direct method with d'Arsonval meter of 100Ω and full scale deflection of 100μA to give the following ranges: 10mV, 1V, and 100V.

Sol:

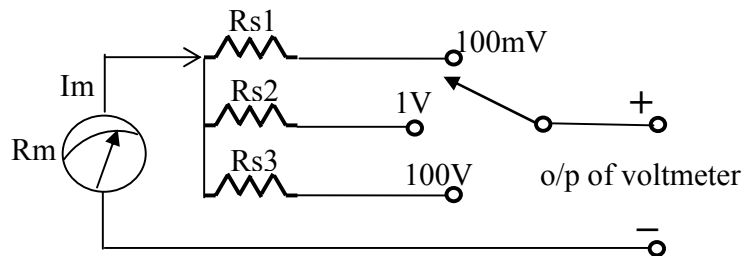
$$R_{s*} = \frac{V_*}{I_m} - R_m$$

$$R_{s1} = \frac{V_1}{I_m} - R_m$$

$$R_{s1} = \frac{10mV}{100\mu A} - 100 = 0\Omega$$

$$R_{s2} = \frac{1}{100 \times 10^{-6}} - 100 = 9.9 K\Omega$$

$$R_{s3} = \frac{100}{100 \times 10^{-6}} - 100 = 99.9 K\Omega$$



3- Ohmmeter and Resistance measurement:

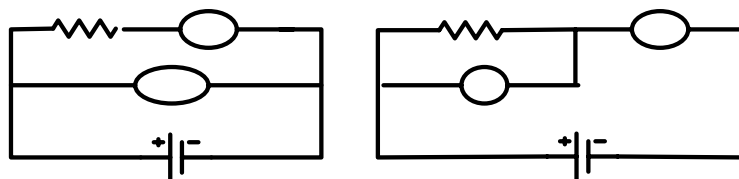
When a current of 1A flows through a circuit which has an impressed voltage of 1 volt, the circuit has a resistance of 1Ω.

$$R = \frac{V}{I}$$

There are several methods used to measure unknown resistance:

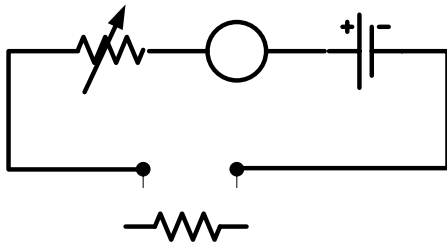
a) Indirect method by ammeter and voltmeter.

This method is inaccurate unless the ammeter has a small resistance and voltmeter have a high resistance.



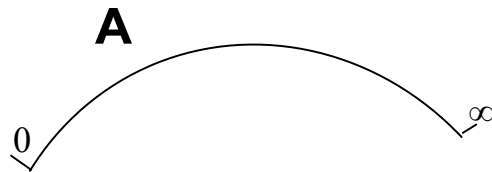
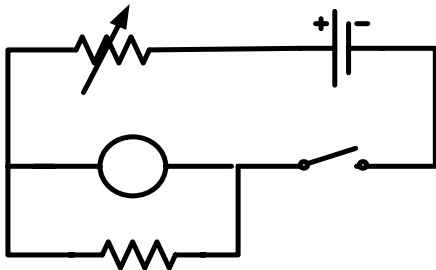
b) Series Ohmmeter:

R_x is the unknown resistor to be measured, R_2 is variable adjusted resistance so that the pointer read zero at short circuit test. The scale of series ohmmeter is nonlinear with zero at the right and infinity at extreme left. Series ohmmeter is the most generally used meter for resistance measurement.



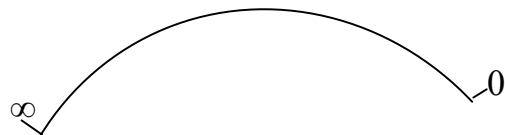
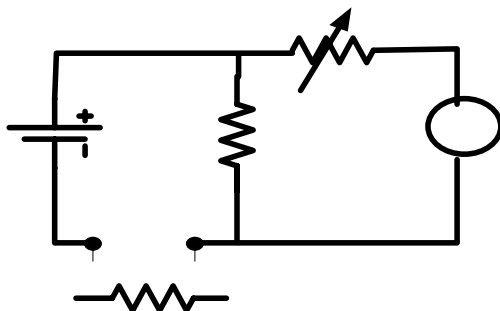
c) Shunt Ohmmeter:

Shunt ohmmeter are used to measure very low resistance values. The unknown resistance R_x is now shunted across the meter, so portion of current will pass across this resistor and drop the meter deflection proportionately. The switch is necessary in shunt ohmmeter to disconnect the battery when the instrument is not used. The scale of shunt ohmmeter is nonlinear with zero at the left and infinity at extreme right.



d) Voltage Divider (potentiometer):

The meter of voltage divider is voltmeter that reads voltage drop across R_s which dependent on R_x . This meter will read from right to left like series ohmmeter with more uniform calibration.



A.c Measuring Instrument

Review on Alternating Signal:

The instantaneous values of electrical signals can be graphed as they vary with time. Such graphs are known as the **waveforms** of the signal. If the value of waveform remains constant with time, the signal is referred to as **direct (d.c)** signal; such as the voltage of a battery. If a signal is time varying and has positive and negative instantaneous values, the waveform is known as **alternating (a.c)** waveform. If the variation of a.c signal is continuously repeated then the signal is known as **periodic** waveform.

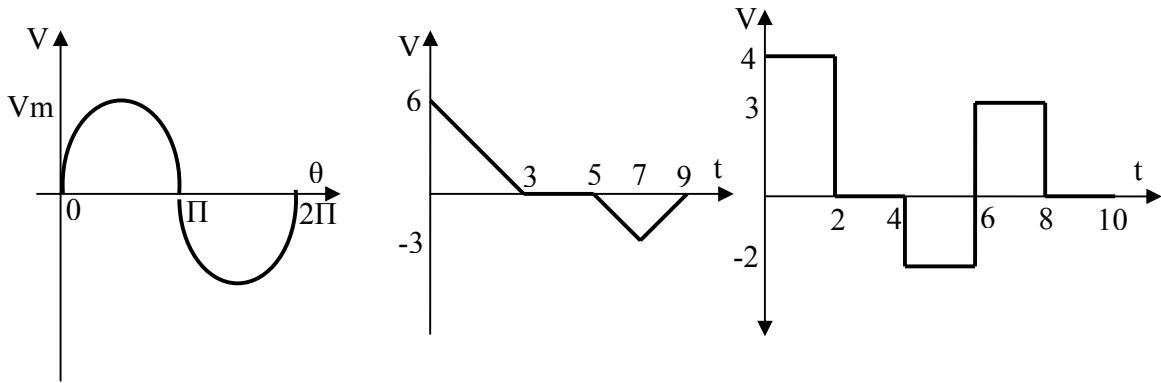
The **frequency of a.c signal** is defined as the *number of cycles traversed in one second*. Thus the time duration of *one cycle per second* for a.c signal is known as the **period (T)**. Where the complete variation of a.c signal before repeated itself is represent one **cycle**.

Average Values:

It is found by dividing the area under the curve of the waveform in one period (T) by the time of the period.

Average value= $\frac{\text{Algebraic sum of the areas under the curve}}{\text{Length of the curve}}$

$$A_v = \frac{\sum \text{areas}}{T} \dots\dots\dots (1) \quad \text{or} \quad A_v = \frac{1}{T} \int_0^T f(t) dt \quad \dots\dots\dots (2)$$



$$A_v = \frac{1}{2\pi} \int_0^{2\pi} V_m \sin \theta d\theta \quad A_v = \frac{\frac{1}{2} \times 3 \times 6 + \frac{1}{2} \times 4 \times (-3)}{9} \quad A_v = \frac{4 \times 2 + (-2) \times 2 + 3 \times 2}{10}$$

$$A_v = -\frac{V_m}{2\pi} (\cos \theta \Big|_0^{2\pi})$$

$$A_v = -\frac{V_m}{2\pi} (1 - 1) = 0$$

The average value for the figure below by using equation (2) is:

$A_v = \frac{1}{T} \int_0^T f(t) dt$ we use the tangent equation for $(x_0, y_0) = (0, 0)$, and $(x_1, y_1) = (3, 6)$ to find the function of $f(t)$

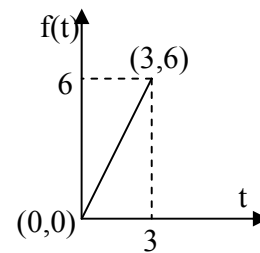
$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow \frac{y - 0}{x - 0} = \frac{6 - 0}{3 - 0} \Rightarrow \frac{y}{x} = \frac{6}{3} = 2 \Rightarrow y = 2x$$

$$f(t) = 2t$$

$$A_v = \frac{1}{3} \int_0^3 (2t) dt$$

$$A_v = \frac{2}{3} \left(\frac{t^2}{2} \Big|_0^3 \right)$$

$$A_v = \frac{1}{3} ((3)^2 - (0)^2) = \frac{9}{3} = 3$$



Root Mean Square Value (effective value of a.c signal):

The r.m.s value of a waveform refers to its power capability. It is refer to the effective value of a.c signal because the r.m.s value equal to the value of a d.c signal which would deliver the same power if it replaced with a.c signal.

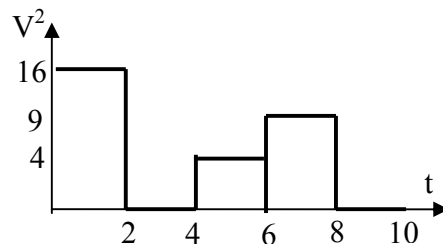
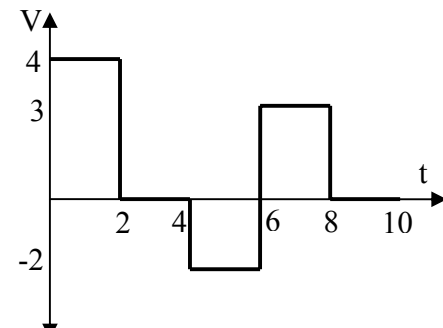
$$r.m.s = \sqrt{\frac{\sum \text{area}(V)^2}{T}} \quad (\text{for square waveform only})$$

$$1- r.m.s = \sqrt{\frac{16 \times 2 + 4 \times 2 + 9 \times 2}{10}}$$

In general form the r.m.s value has the following aqua.

$$r.m.s = \sqrt{\text{Average } f(t)^2}$$

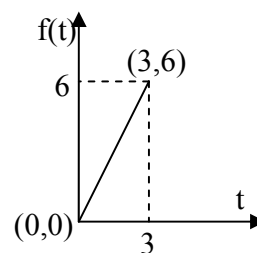
$$r.m.s = \sqrt{\frac{1}{T} \int_0^T f(t)^2 dt}$$



2- If $f(t) = 2t$ then its r.m.s value is:

$$r.m.s = \sqrt{\frac{1}{3} \int_0^3 (2t)^2 dt}$$

$$r.m.s = \sqrt{\frac{4}{3} \left(\frac{t^3}{3} \Big|_0^3 \right)} = \sqrt{\frac{4}{9} ((3)^3 - (0)^3)} = \sqrt{\frac{4 \times 27}{9}} = 3.46$$



3- If $f(t) = V_m \sin \theta$

$$r.m.s = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_m^2 \sin^2 \theta d\theta}$$

$$r.m.s = \sqrt{\frac{V_m^2}{2\pi} \int_0^{2\pi} \frac{1 - \cos 2\theta}{2} d\theta}$$

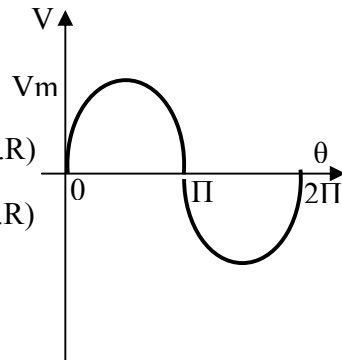
$$r.m.s = \left\{ \frac{V_m^2}{4\pi} \left[\int_0^{2\pi} d\theta - \int_0^{2\pi} \cos 2\theta d\theta \right] \right\}^{\frac{1}{2}} \quad r.m.s = \sqrt{\frac{V_m^2}{4\pi} \left[\theta \Big|_0^{2\pi} - \frac{1}{2} \sin 2\theta \Big|_0^{2\pi} \right]}$$

$$r.m.s = \sqrt{\frac{Vm^2}{4\pi} [2\pi - 0]} = \sqrt{\frac{Vm^2}{2}} = \frac{Vm}{\sqrt{2}}$$

$$\text{FormFactor} = \frac{r.m.s}{\text{average}} \quad \text{for Sine wave F.F}=1.11 \text{ (F.W.R)}$$

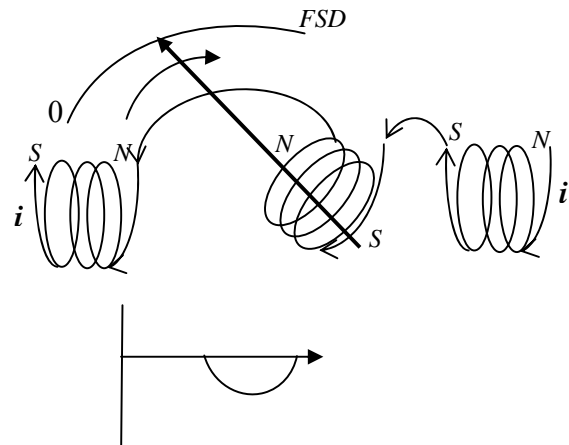
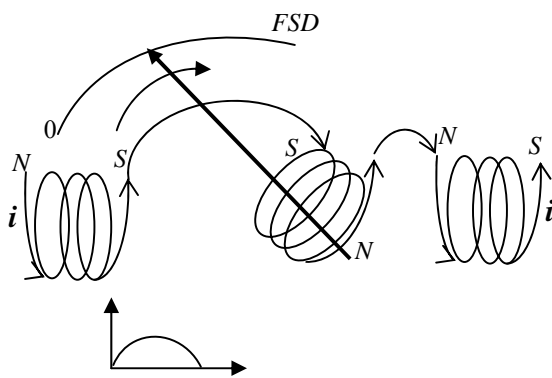
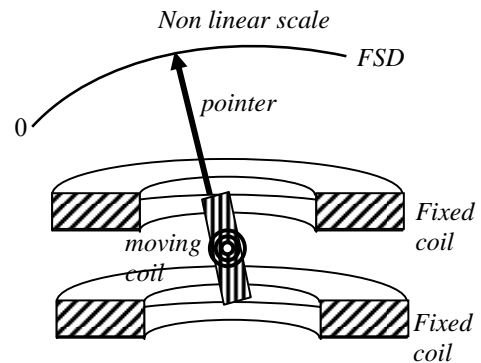
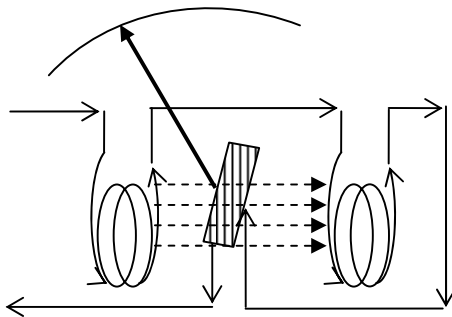
$$\text{CrestFactor} = \frac{\text{PeakValue}}{r.m.s}$$

$$\text{F.F}=1.57 \text{ (H.W.R)}$$



Dynamometer:

This instrument is suitable for the measurement of direct and alternating current, voltage and power. The deflecting torque in dynamometer is relies by the interaction of magnetic field produced by a pair of fixed air cored coils and a third air cored coil capable of angular movement and suspended within the fixed coil.

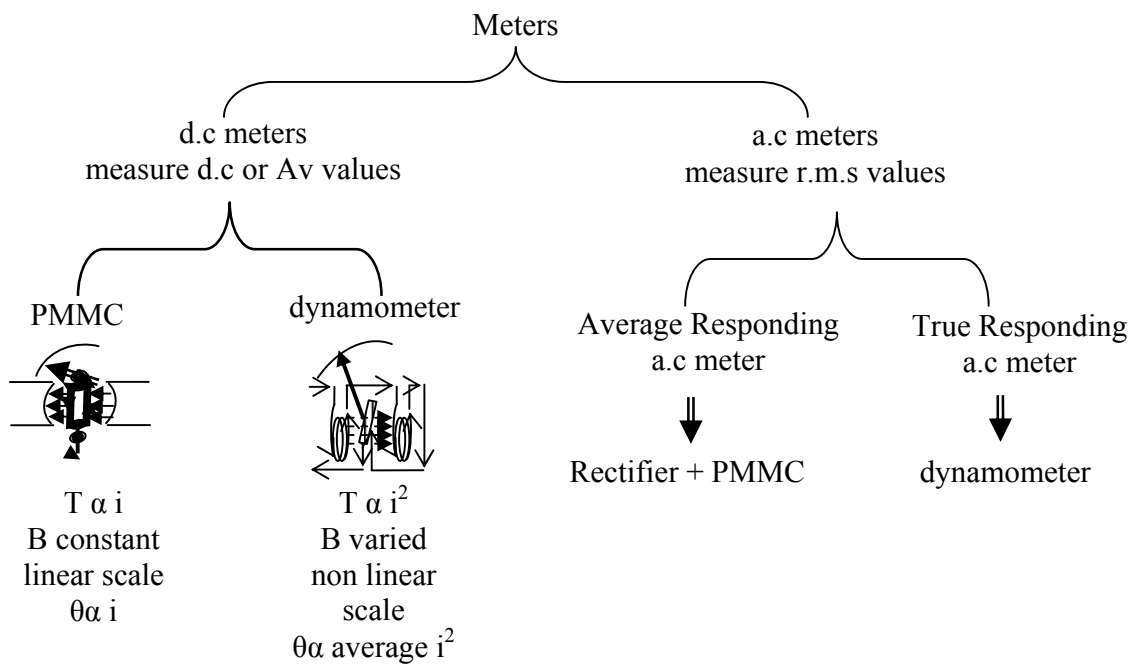
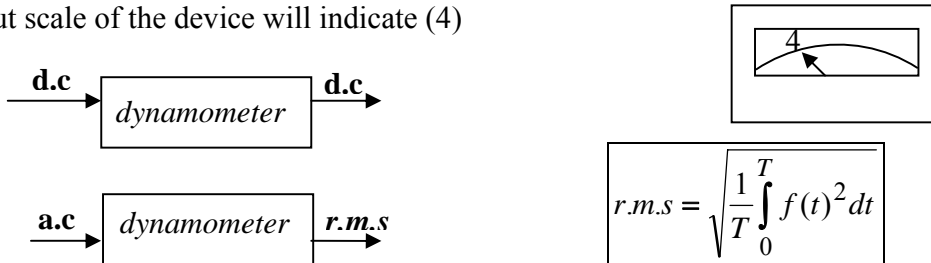


$$T_i = N \vec{B} i_m A, \quad \vec{B} \propto i_f \quad \text{thus} \quad T_i \propto i_m i_f A \implies \quad \text{so } T_i \propto i^2$$

$$\theta \propto \text{average } i^2, \quad \text{since} \quad r.m.s = \sqrt{\text{average } f(t)^2}$$

The **output scale is calibrated to give the r.m.s value of a.c signal** by taking the square roots of the inside measured value.

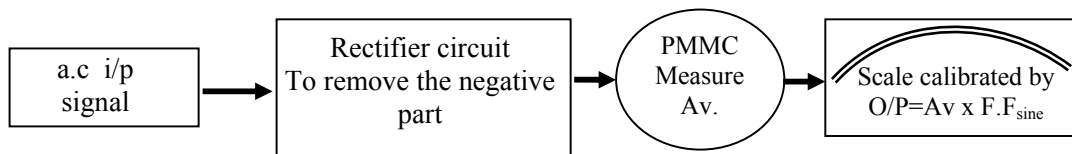
O/P scale = $r.m.s = \sqrt{\text{average}(i)^2}$, for example if $(\text{average } i^2) = 16$ inside the measuring device, the output scale of the device will indicate (4)



1-Average Responding a.c Meter:

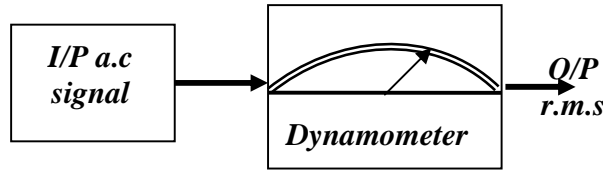
O/P (r.m.s) = $A_v \times F.F_{\text{sine wave}}$ (measured) $F.F_{\text{sine wave}}(\text{F.W.R}) = 1.11$
 $F.F_{\text{sine wave}}(\text{H.W.R}) = 1.57$

O/P (r.m.s) = $A_v \times F.F_{\text{true}}$ (true) $F.F_{\text{true}} =$ The form factor of any input signal (sine, square, or any thing)



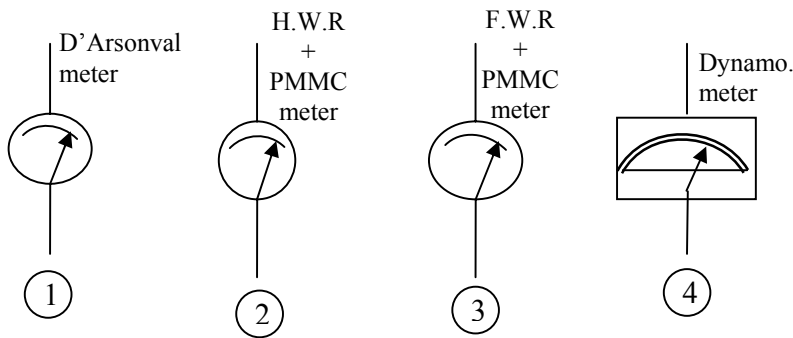
2-True Responding a.c Meter (Dynamometer):

$O/P (r.m.s) = A_v \times F.F_{true}$ $F.F_{true} =$ The form factor of any input signal
 (true) = (measured)



Example:

What will be the out put of the following meters, if an average responding a.c meter of half-wave rectifier read (4.71v), and true form factor of input waveform is (1.414).



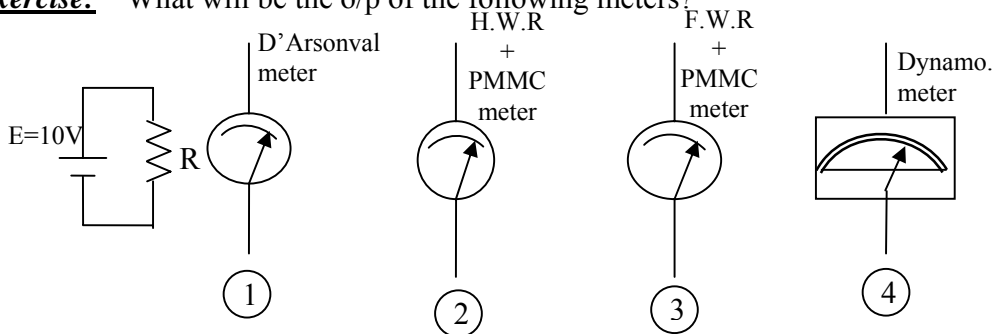
Sol:

$r.m.s_{measured} = 1.57 \times A_v$ for average responding a.c meter of half wave rectifier

$4.71 = 1.57 \times A_v \Rightarrow A_v = \frac{4.71}{1.57} = 3V$

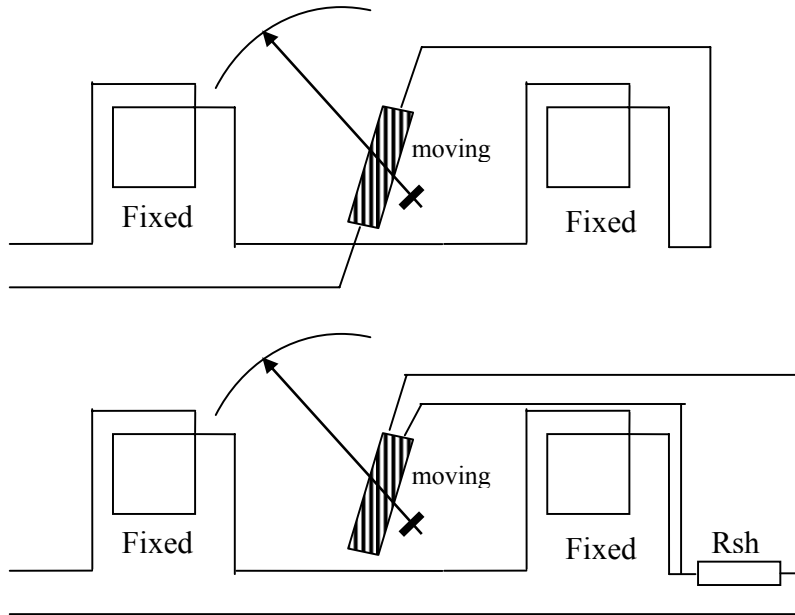
1. D'Arsonval meter read $A_v = 3V$
2. HWR+PMMC (**Average responding of halve wave rectifier**) meter = 4.71V
3. FWR+PMMC (**Average responding of full wave rectifier**) meter = $1.11 \times 3 = 3.33V$
4. Dynamometer = $F.F_{(true)} \times A_v$
 $r.m.s_{(true)} = 1.414 \times 3 = 4.242V$

Exercise: What will be the o/p of the following meters?

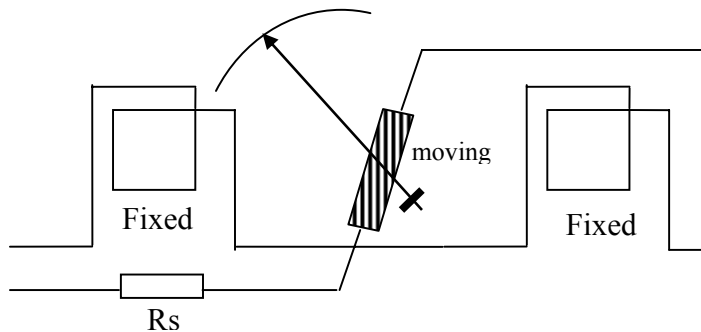


Dynamometer As Ammeter And Voltmeter:

For small current measurement (5mA to 100mA), fixed and moving coils are connect in series. While larger current measurement (up to 20A) , the moving coil is shunted by a small resistance.

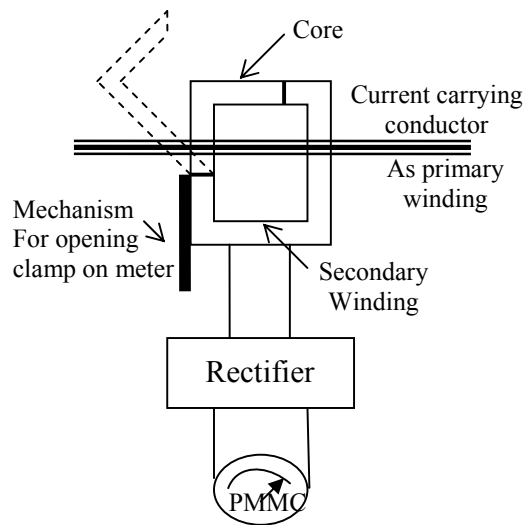


To convert such an instrument to a voltmeter only a rather big series resistance is connected with the moving coil.



Clamp on Meters (Average Responding A.C meter):

One application of average responding a.c meters is the **clamp on meter** which is used to measured a.c current, voltage in a wire **with out having to break** the circuit being measured. The meter having use the transformer principle to detect the current. That is, the clamp on device of the meter serves as the core of a transformer. The current carrying wire is the primary winding of the transformer, while the secondary winding is in the meter. The alternating current in the primary is coupled to the secondary winding by the core, and after being rectified the current is sensed by a d'Arsonval meter.



Example:

The symmetrical square wave voltage is applied to an average responding a.c voltmeter with a scale calibrated in term of the r.m.s value of a sine wave. Calculate:

1. The form factor of square wave voltage.
2. The error in the meter indication.

Sol:

$$V_{rms(True)} = \sqrt{\frac{1}{T} \int_0^T V(t)^2 dt} = V_m$$

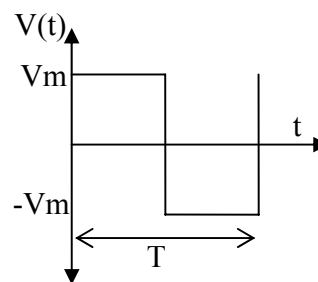
$$V_{average(True)} = \frac{2}{T} \int_0^{\frac{T}{2}} V(t) dt = V_m$$

$$F.F(True) = \frac{V_{rms}}{V_{av.}} = \frac{V_m}{V_m} = 1$$

$$V_{rms(measured)} = 1.11 \times Av. = 1.11 \times V_m = 1.11V_m$$

$$Error = \frac{V_{rms(True)} - V_{rms(measured)}}{V_{rms(True)}} \times 100\%$$

$$Error = \frac{V_m - 1.11V_m}{V_m} \times 100\%$$



Exer.:

Repeat the above example for saw tooth waveform shown

Sol:

$$V(t) = 25t$$

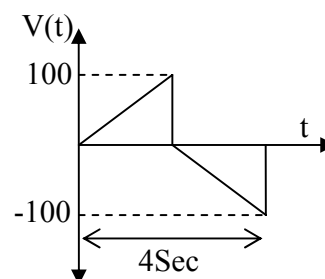
$$V_{av.} = 50V$$

$$V_{rms(True)} = 57.75V$$

$$V_{rms(Measured)} = 55.5V$$

$$F.F(Tru) = 1.154$$

$$Error = 0.0389\%$$



Bridges and Their Application

Bridge circuit are extensively used for **measuring component values**, such as *resistance, inductance, capacitance*, and other circuit parameters directly derived from component values such as *frequency, phase angle, and temperature*. Bridge accuracy measurements are very high because their circuit merely compares the value of an unknown component to that of an accurately known component (a standard).

1- D.c Bridges:

The basic d.c bridges consist of four resistive arms with a source of emf (a battery) and a null detector usually galvanometer or other sensitive current meter. D.c bridges are generally used for the measurement of resistance values.

a) Wheatstone Bridge:

This is the best and commonest method of measuring **medium** resistance values in the range of 1Ω to the low megohm. The current through the galvanometer depends on potential difference between point (c) and (d). The **bridge** is said to be **balance** when potential difference across the galvanometer is zero volts, so there is no current through the galvanometer ($I_g=0$). This condition occurs when $V_{ca}=V_{da}$ or $V_{cb}=V_{db}$ hence the bridge is balance when

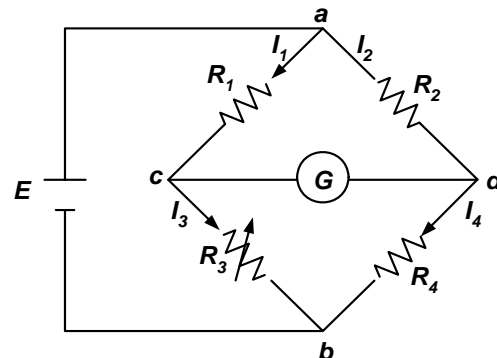
$V_1 = V_2$ (1) Since $I_g = 0$ so by voltage divider rule

$$V_1 = E \frac{R_1}{R_1 + R_3} \text{ (2) and}$$

$$V_2 = E \frac{R_2}{R_2 + R_4} \text{ (3)}$$

Substitute equations (2) & (3) in equ. (1)

$$\frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4}$$



Thus $R_1 R_4 = R_2 R_3$ is the balance equation for Wheatstone bridge

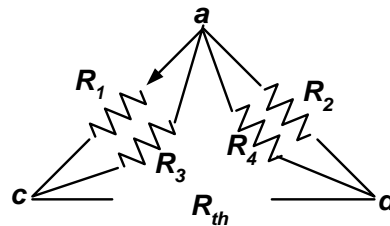
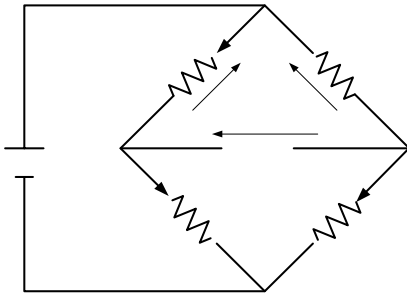
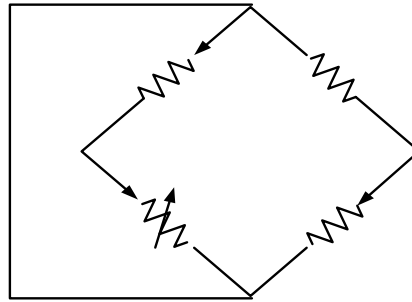
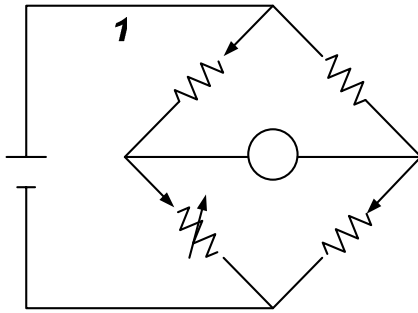
So, if three of resistance values are known, the fourth unknown ones can be determined.

$$R_4 = \frac{R_3 R_2}{R_1}$$

R_3 are called the standard arm of the bridge and resistors R_2 and R_1 are called the ratio arms.

Thevenin Equivalent Circuit:

To determine whether or not the galvanometer has the required sensitivity to detect an unbalance condition, it is necessary to calculate the galvanometer current for small unbalance condition. The solution is approached by converting the Wheatstone bridge to its thevenin equivalent. Since we are interested in the current through the galvanometer, the thevenin equivalent circuit is determined by looking into galvanometer terminals (c) and (d).



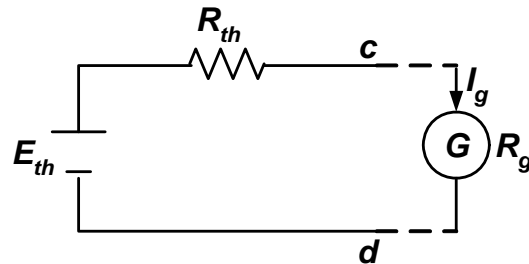
when $E = 0$

$$R_{th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$E_{th} = V_1 - V_2 \quad E \neq 0$$

$$E_{th} = \frac{ER_1}{R_1 + R_3} - \frac{ER_2}{R_2 + R_4}$$

$$I_g = \frac{E_{th}}{R_{th} + R_g}$$



and galvanometer deflection (d) is:

$$d = I_g \times \text{current sensitivity (mm/}\mu\text{A)}$$

b) Kelvin Bridge:

Kelvin bridge is a modification of the Wheatstone bridge and provides greatly increased accuracy in the measurement of **low value** resistance, generally below (1Ω) . It is eliminate errors due to contact and leads resistance. (R_y) represent the resistance of the connecting lead from R_3 to R_4 . Two galvanometer connections are possible, to point (m) or to point (n).

1- If the galvanometer connect to point (m) then

$R_4 = R_x + R_y$ therefore unknown resistance will be higher than its actual value by R_y

2- If the galvanometer connect to point (n) then

$R_4 = R_3 + R_y$ therefore unknown resistance will be lower than its actual value by R_y

3- If the galvanometer connect to point (p) such that

$$\frac{R_{np}}{R_{mp}} = \frac{R_1}{R_2} \dots\dots\dots (1)$$

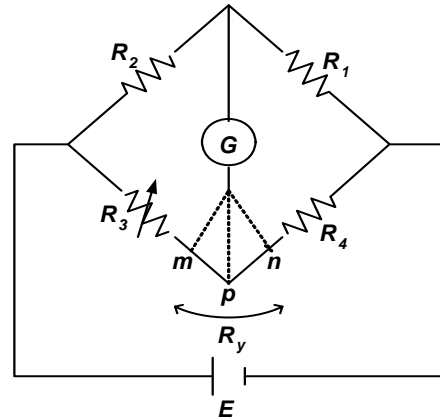
At balance condition

$$R_2(R_x + R_{np}) = R_1(R_3 + R_{mp}) \dots\dots\dots (2)$$

Substituting equ.(1) in to equ.(2) we obtain

$$R_x + \left(\frac{R_1}{R_1 + R_2}\right)R_y = \frac{R_1}{R_2} \left[R_3 + \left(\frac{R_2}{R_1 + R_2}\right)R_y \right]$$

This reduces to
$$R_x = \frac{R_1}{R_2} R_3$$



So the effect of the resistance of the connecting lead from point (m) to point (n) has be eliminated by connecting the galvanometer to the intermediate position (p).

c) Kelvin Double Bridge:

Kelvin double bridge is used for measuring **very low** resistance values from approximately (1Ω to as low as 1x10⁻⁵Ω). The term double bridge is used because the circuit contains a second set of ratio arms labelled Ra and Rb. If the galvanometer is connect to point (p) to eliminates the effect of (yoke resistance Ry).

$$\frac{R_a}{R_b} = \frac{R_1}{R_2}$$

At balance $V_2 = V_3 + V_b \dots\dots\dots (1)$

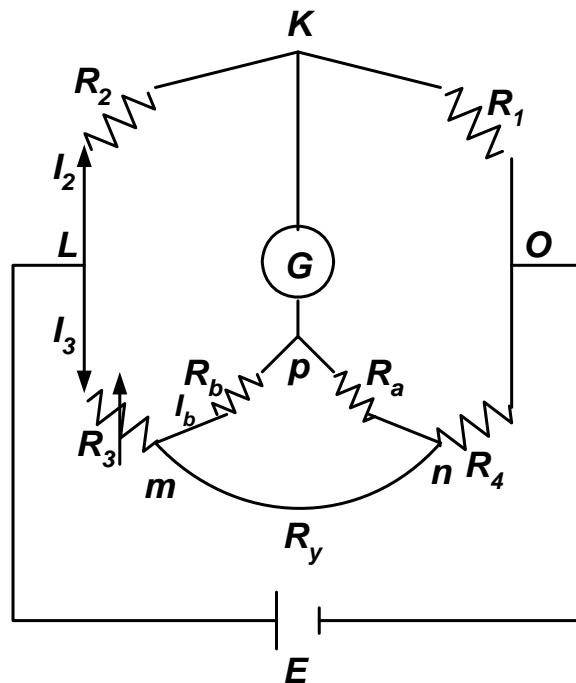
$$V_2 = E \frac{R_2}{R_1 + R_2} \dots\dots\dots (2)$$

$$V_3 = I_3 R_3 \quad \text{and} \quad V_b = I_b R_b \dots\dots (3)$$

$$I_b = I_3 \frac{R_y}{(R_a + R_b) + R_y} \dots\dots\dots (4)$$

$$E = I_3 \left[R_3 + \frac{(R_a + R_b)R_y}{(R_a + R_b) + R_y} + R_4 \right] \dots (5)$$

Sub.equ. (5) in to equ. (2) and equ. (4) into equ.(3) then substitute the result in equ.(1), we get



$$I_3 \left[R_3 + \frac{(R_a + R_b)R_y}{(R_a + R_b) + R_y} + R_4 \right] \frac{R_2}{R_1 + R_2} = I_3 R_3 + I_3 \frac{R_y}{(R_a + R_b) + R_y} R_b$$

$$R_x = \frac{R_3 R_1}{R_2} + \frac{R_y R_b}{R_a + R_b + R_y} \left[\frac{R_1}{R_2} + 1 - 1 - \frac{R_a}{R_b} \right]$$

$$R_x = \frac{R_3 R_1}{R_2} + \frac{R_y R_b}{R_a + R_b + R_y} \left[\frac{R_1}{R_2} - \frac{R_a}{R_b} \right]$$

This is the balanced equation

If $\frac{R_a}{R_b} = \frac{R_1}{R_2}$ then $R_x = \frac{R_3 R_1}{R_2}$

2-Ac Bridge and Their Application:

The ac bridge is a natural outgrowth of the dc bridge and in its basic form consists of four bridge arms, a source of excitation, and a null ac detector. For measurements at low frequencies, the power line may serve as the source of excitation; but at higher frequencies an oscillator generally supplies the excitation voltage. The null ac detector in its cheapest effective form consists of a pair of headphones or may be oscilloscope.

The balance condition is reached when the detector response is zero or indicates null. Then $V_{AC} = 0$ and $V_{Z1} = V_{Z2}$

$$V_{Z1} = V_{in} \frac{Z_1}{Z_1 + Z_3}$$

$$V_{Z2} = V_{in} \frac{Z_2}{Z_2 + Z_4} \quad \text{thus}$$

$$Z_1 Z_4 = Z_2 Z_3 \quad \text{is the balance equation}$$

$$\text{Or } Y_1 Y_4 = Y_2 Y_3$$

The balance equation can be written in complex form as:

$$(Z_1 \angle \theta_1)(Z_4 \angle \theta_4) = (Z_2 \angle \theta_2)(Z_3 \angle \theta_3)$$

$$\text{And } (Z_1 Z_4 \angle \theta_1 + \theta_4) = (Z_2 Z_3 \angle \theta_2 + \theta_3)$$

So two conditions must be met simultaneously when balancing an ac bridge

$$1- Z_1 Z_4 = Z_2 Z_3$$

$$2- \angle \theta_1 + \angle \theta_4 = \angle \theta_2 + \angle \theta_3$$

Review on Ac Impedance:

a) In series connection

Impedance = resistance \pm j reactance

$$Z_L = R + jXL \quad \text{and} \quad Z_L = R + j\omega L$$

$$Z_C = R - jXC \quad \text{and} \quad Z_C = R - j \frac{1}{\omega C}$$

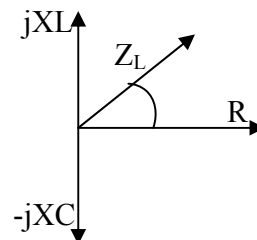
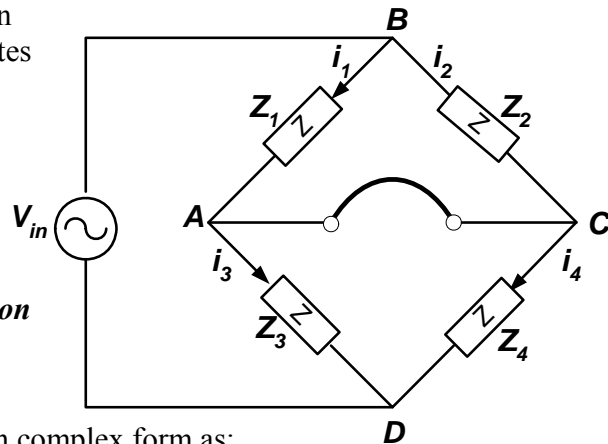
Conversion from polar to rectangular

$$Z \angle \theta \quad \text{in polar form} \quad R = Z \cos \theta$$

$$X = Z \sin \theta \quad \text{become } Z = R \pm jX$$

Conversion from rectangular to polar

$$Z = R \pm jX \quad \text{in rectangular form} \quad Z = \sqrt{R^2 + X^2} \quad \theta = \tan^{-1} \frac{X}{R} \quad \tan \theta = \frac{X}{R}$$



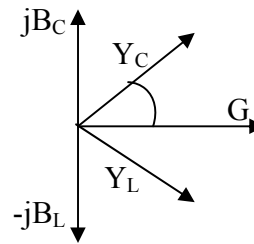
b) **In parallel connection**

Admittance = conductance ± j susceptance

$$Y_L = G - jB_L \quad \text{and} \quad Y_L = \frac{1}{R} - j \frac{1}{\omega L}$$

$$Y_C = G + jB_C \quad \text{and} \quad Y_C = \frac{1}{R} + j\omega C$$

$$\tan \theta = \frac{B_C}{G} = \frac{\frac{1}{X_C}}{\frac{1}{R}} = \frac{\omega C}{\frac{1}{R}} = \omega RC$$



Example (1):

The impedance of the basic a.c bridge are given as follows:

$$Z_1 = 100 \angle 80^\circ \quad (\text{inductive impedance}) \quad Z_2 = 250 \Omega \quad Z_3 = 400 \angle 30^\circ \quad (\text{inductive impedance})$$

$Z_4 = \text{unknown}$

Sol:

$$\boxed{Z_4 = \frac{Z_2 Z_3}{Z_1}} \quad Z_4 = \frac{250 \times 400}{100} = 1k\Omega \quad \boxed{\theta_4 = \theta_2 + \theta_3 - \theta_1} \quad \theta_4 = 0 + 30 - 80 = -50^\circ$$

$Z_4 = 1000 \angle -50^\circ$ (capacitive impedance)

Example (2):

For the following bridge find Z_x ?

The balance equation $Z_1 Z_4 = Z_2 Z_3$

$$Z_1 = R = 450 \Omega$$

$$Z_2 = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$$

$$Z_2 = 300 - j600$$

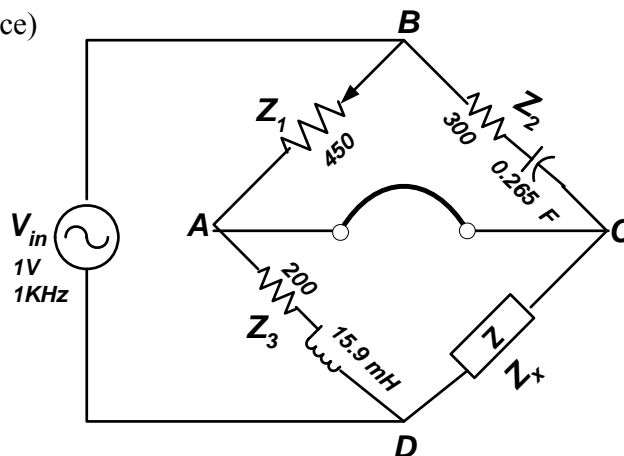
$$Z_3 = R + j\omega L$$

$$Z_3 = 200 + j100$$

$$Z_4 = Z_x = \text{unknown}$$

$$Z_4 = \frac{Z_2 Z_3}{Z_1} \quad Z_4 = \frac{(300 - j600)(200 + j100)}{450} = 266.6 - j200$$

$$R = 266.6 \Omega \quad C = \frac{1}{2\pi f \times 200} = 0.79 \mu F$$



a) Comparison Bridges:

A.c comparison bridges are used to measure unknown inductance or capacitance by comparing it with a known inductance or capacitance.

1- Capacitive Comparison Bridge:

In capacitive comparison bridge R_1 & R_2 are ratio arms, R_s in series with C_s are standard known arm, and C_x represent unknown capacitance with its leakage resistance R_x .

$$Z_1 = R_1 \quad Z_2 = R_2 \quad Z_3 = R_s - \frac{j}{\omega C_s} \quad Z_4 = R_x - \frac{j}{\omega C_x}$$

At balance $\boxed{Z_1 Z_4 = Z_2 Z_3}$

$$R_1 \left(R_x - \frac{j}{\omega C_x} \right) = R_2 \left(R_s - \frac{j}{\omega C_s} \right)$$

$$R_1 R_x - \frac{j R_1}{\omega C_x} = R_2 R_s - \frac{j R_2}{\omega C_s}$$

By equating the real term with the real and imaginary term with imaginary we get:

$$R_1 R_x = R_2 R_s \quad \boxed{R_x = \frac{R_2 R_s}{R_1}}$$

$$\frac{-j R_1}{\omega C_x} = \frac{-j R_2}{\omega C_s} \quad \boxed{C_x = \frac{R_1 C_s}{R_2}}$$

We can **note** that the bridge is **independent** on **frequency** of applied source.

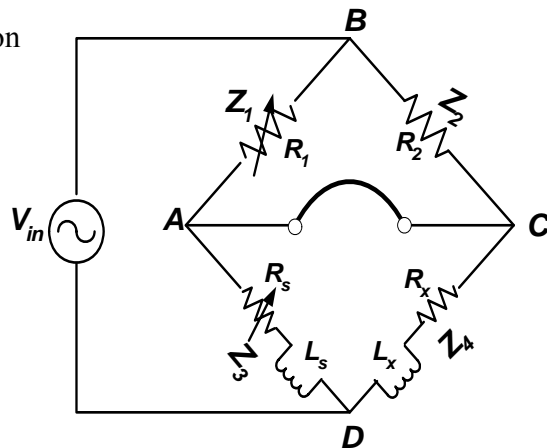
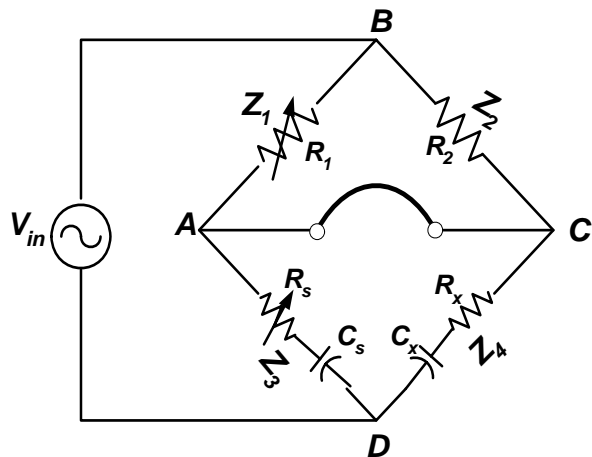
2- Inductive Comparison Bridge:

The unknown inductance is determined by comparing it with a known standard inductor.

At balance we get

$$\boxed{R_x = \frac{R_2 R_s}{R_1}} \text{ represent resistive balance equation}$$

$$\boxed{L_x = \frac{R_2 L_s}{R_1}} \text{ inductive balance equation}$$



b) Maxwell bridge:

This bridge measure **unknown inductance** in terms of **a known capacitance**, at balance:

$$Z_1 Z_4 = Z_2 Z_3 \quad Z_1 = \frac{1}{Y_1} \text{ thus}$$

$$\boxed{Z_4 = Z_2 Z_3 Y_1} \text{ where}$$

$$Z_2 = R_2 \quad Z_3 = R_3 \quad Y_1 = \frac{1}{R_1} + j\omega C_1$$

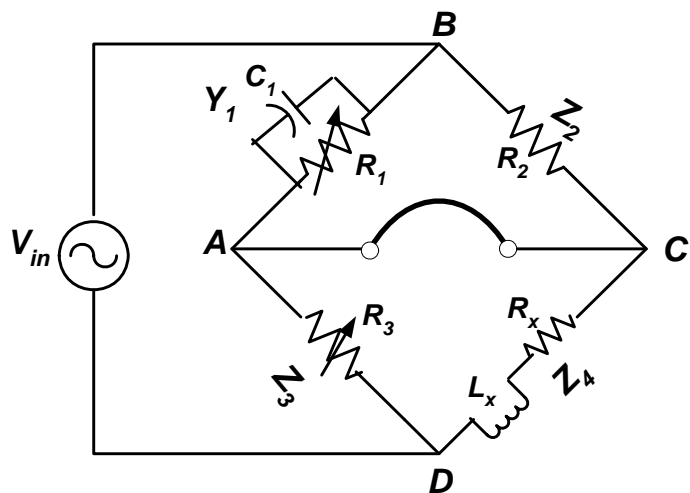
$$Z_4 = R_x + j\omega L_x$$

So

$$R_x + j\omega L_x = R_2 R_3 \left(\frac{1}{R_1} + j\omega C_1 \right)$$

$$R_x = \frac{R_2 R_3}{R_1}$$

$$L_x = R_2 R_3 C_1$$



Maxwell bridge is limited to the measurement of *medium quality factor (Q) coil* with range between $1 < Q \leq 10$

$$|\tan \theta_1| = |\tan \theta_4| = \frac{\omega L_4}{R_4} = \frac{B_{c1}}{G_1} = \frac{XC_1}{\frac{1}{R_1}} = \omega R_1 C_1 = Q$$

c) Hay Bridge:

Hay bridge convening for *measuring high Q coils*

$$Z_1 = R_1 - \frac{j}{\omega C_1} \quad Z_2 = R_2 \quad Z_3 = R_3$$

$$Z_4 = R_x + j\omega L_x$$

At balance $Z_1 Z_4 = Z_2 Z_3$

$$\left(R_1 - \frac{j}{\omega C_1} \right) (R_x + j\omega L_x) = R_2 R_3$$

$$R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega R_1 L_x = R_2 R_3$$

Separating the real and imaginary terms

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \dots\dots (1)$$

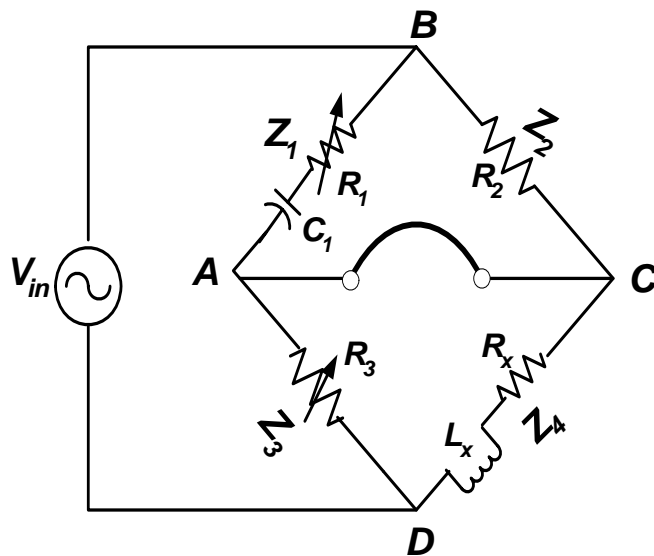
$$\frac{R_x}{\omega C_1} = \omega R_1 L_x \dots\dots\dots (2)$$

Solving equ.(1) and (2) yields

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + \omega^2 C_1^2 R_1^2}$$

$$L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 C_1^2 R_1^2}$$

$$\theta_1 = -\theta_4 \text{ because } \theta_2 = \theta_3 = \text{zero}$$



$$|\tan \theta_1| = |\tan \theta_4| = \frac{\omega L_4}{R_4} = \frac{XC_1}{R_1} = \frac{\omega C_1}{\frac{1}{R_1}} = \frac{1}{\omega C_1 R_1} = Q$$

Thus $Q = \frac{1}{\omega R_1 C_1} \dots\dots\dots (3)$

Submitted equ.(3) in to equ. (2) yield

$$L_x = \frac{R_2 R_3 C_1}{1 + \left(\frac{1}{Q}\right)^2} \quad \text{For } Q > 10, \text{ then } \left(\frac{1}{Q}\right)^2 \ll 1 \text{ and can be neglected, then } L_x = R_2 R_3 C_1$$

d) Schering Bridge:

Schering bridge used extensively for capacitive measurement, (C3) is standard high mica capacitor for general measurement work, or (C3) may be an air capacitor for insulation measurements. The balance condition require that $\theta_1 + \theta_4 = \theta_2 + \theta_3$ but $\theta_1 + \theta_4 = -90$ Thus $\theta_2 + \theta_3$ must equal (-90) to get balance

At balance $Z_4 = Z_2 Z_3 Y_1$

$$Y_1 = \frac{1}{R_1} + j\omega C_1 \quad Z_2 = R_2 \quad Z_3 = \frac{-j}{\omega C_3}$$

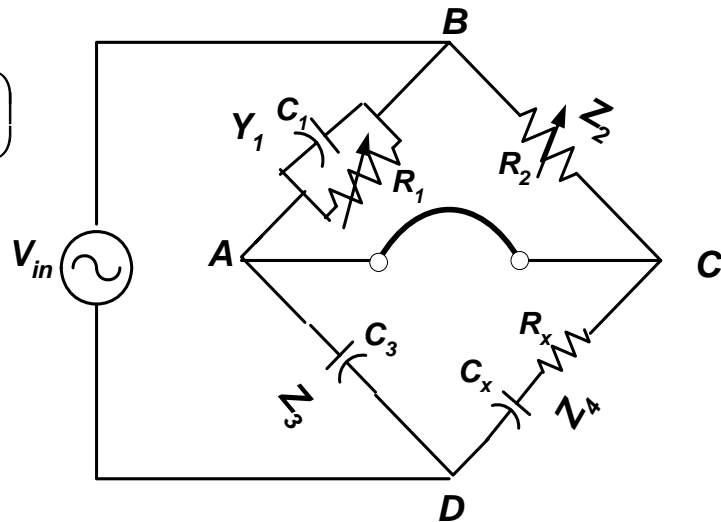
$$Z_4 = R_x - \frac{j}{\omega C_x}$$

$$R_x - \frac{j}{\omega C_x} = R_2 \left(\frac{-j}{\omega C_3} \right) \left(\frac{1}{R_1} + j\omega C_1 \right)$$

$$R_x - \frac{j}{\omega C_x} = \frac{R_2 C_1}{C_2} - \frac{j R_2}{\omega C_3 R_1}$$

$$R_x = R_2 \frac{C_1}{C_3} \dots\dots (1)$$

$$C_x = C_3 \frac{R_1}{R_2} \dots\dots (2)$$



The power factor (pf):

$$pf = \cos \theta_c = \frac{R_x}{Z_x}$$

The dissipation factor (D):

$$D = \cot \theta_c = \frac{R_x}{X C_x} = \frac{1}{Q} = \omega R_x C_x \dots\dots (3)$$

Substitute eqs. (1) & (2) into (3), we get

$$D = \omega R_1 C_1$$

e) Wien Bridge:

This bridge is used to measured *unknown frequency*

$$Z_1 = R_1 - \frac{j}{\omega C_1} \quad Z_2 = R_2 \quad Y_3 = \frac{1}{R_3} + j\omega C_3 \quad Z_4 = R_4$$

$$Z_1 Z_4 = \frac{Z_2}{Y_3}$$

$$Z_2 = Z_1 Z_4 Y_3$$

$$R_2 = \left(R_1 - \frac{j}{\omega C_1} \right) R_4 \left(\frac{1}{R_3} + j\omega C_3 \right)$$

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1}$$

Dividing by R4 we get

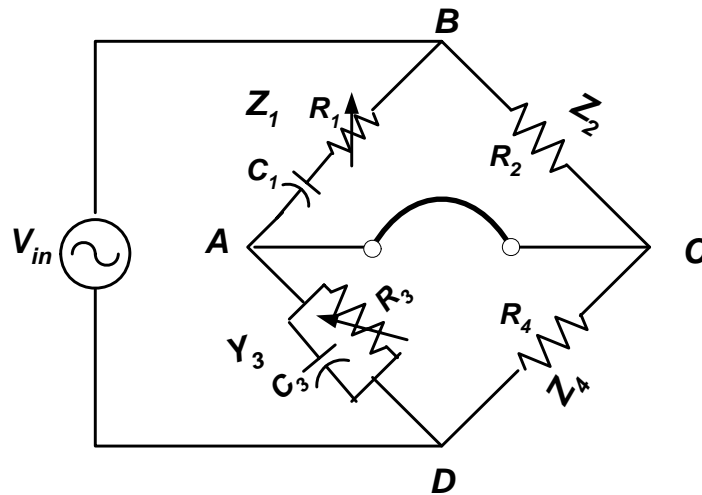
$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \dots\dots (1)$$

Equating the imaginary terms, yield

$$\omega C_3 R_1 R_4 = \frac{R_4}{\omega C_1 R_3} \quad \text{Since } \omega = 2\pi F$$

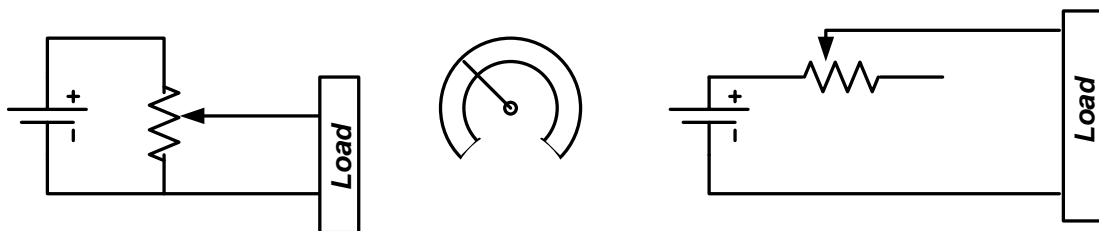
Thus $F = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}}$ if $R_1 = R_3$ and $C_1 = C_3$ then $\frac{R_2}{R_4} = 2$ in equ.(1)

And $F = \frac{1}{2\pi RC}$ *this is the general equation for Wien bridge*



Variable Resistors:

The variable resistance usually have three leads, two fixed and one movable. If the contacts are made to only **two leads** of the resistor (**stationary lead and moving lead**), the variable resistance is being employed as a **rheostat** which **limit the current flowing** in circuit branches. If all **three contacts** are used in a circuit, it is termed a **potentiometer** or pot and often used as **voltage dividers** to control or vary voltage across a circuit branch.



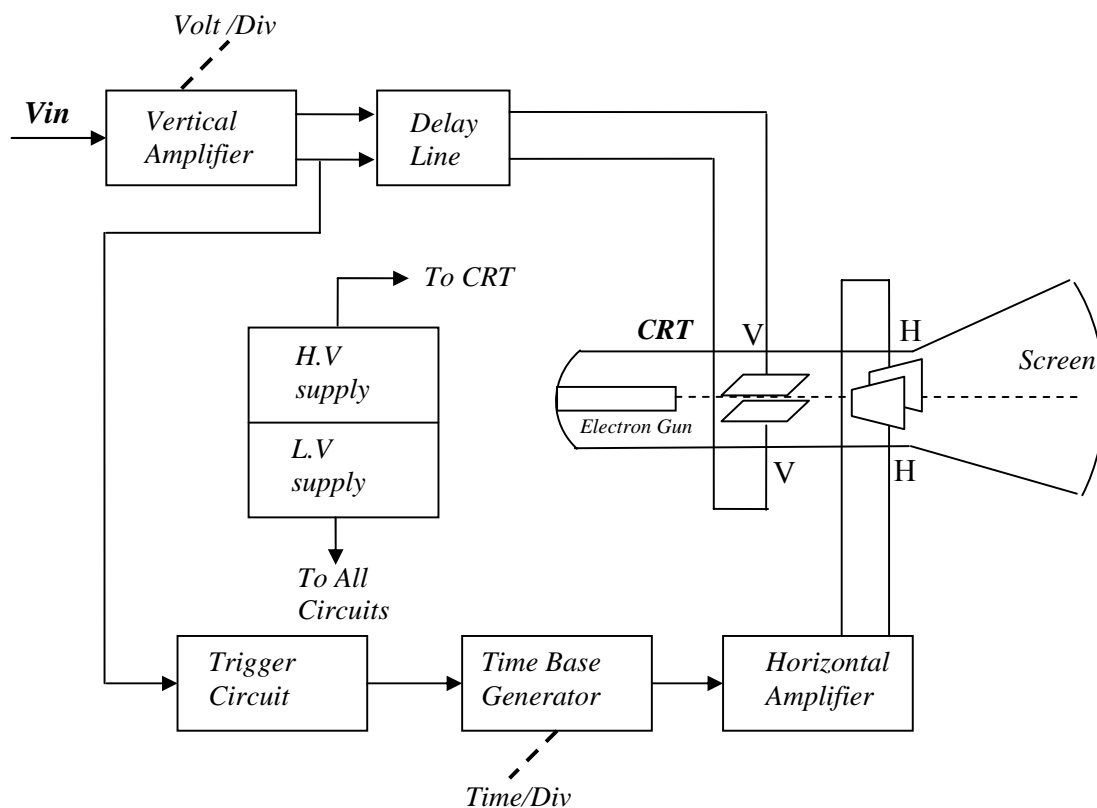
Oscilloscope

The *cathode ray oscilloscope (CRO)* is a device that allows the *amplitude* of electrical signals, whether they are voltage, current; power, etc., to be displayed primarily as a function of *time*. The oscilloscope depends on the movement of an electron beam, which is then made visible by allowing the beam to impinge on a phosphor surface, which produces a visible spot

Oscilloscope Block Diagram:

General oscilloscope consists of the following parts:

1. Cathode ray tube (CRT)
2. Vertical deflection stage
3. Horizontal deflection stage
4. Power supply



General Purpose Oscilloscope

The Cathode Ray Tube (CRT):

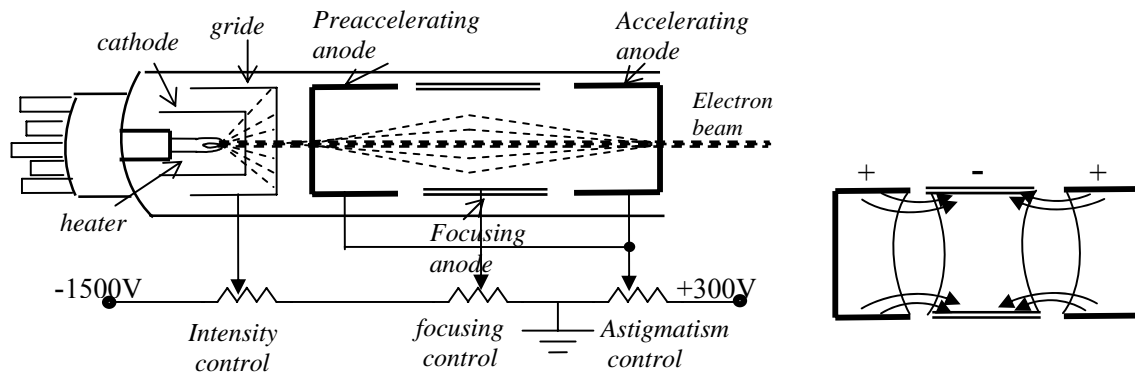
Cathode ray tube is the *heart* of oscilloscope which *generates* the electron beam, *accelerates* the beam to high velocity, *deflects* the beam to create the image, and contains the phosphor screen where the electron beam eventually *become visible*. There are two standard type of CRT *electromagnetic and electrostatic*. Each CRT contains:

- a) One or more electron guns.
- b) Electrostatic deflection plates.
- c) Phosphoresce screen.

A **gun** consists of a **heated cathode, control grid, and three anodes**.

A **heated cathode** emits electrons, which are accelerated to the first accelerating anode, through a small hole in the **control grid**. The amount of cathode current, which governs the **intensity** of the spot, can be controlled with the control grid. The **preaccelerating anode** is a hollow cylinder that is at potential a few hundred volts more positive than the cathode so that the electron beam will be accelerated in the electric field. A **focusing anode** is mounted just ahead of the preaccelerating anode and is also a cylinder. Following the focusing anode is the **accelerating anode**, which gives the electron beam its last addition of energy before its journey to the deflecting plates. The focusing and accelerating anodes form an electrostatic lens, which bring the electron beam into spot focus on the screen. Three controls are associated with the operating voltages of the CRT; **intensity, focus, and astigmatism**

- 1- The intensity control varies the potential between the cathode and the control grid and simply adjusts the beam current in the tube.
- 2- The focus control adjusts the focal length of the electrostatic lens.
- 3- The astigmatism control adjusts the potential between the deflection plates and the first accelerating electrode and is used to produce a round spot.



The **electrostatic deflection** system consists of **two sets of plates** for each electron gun. The **vertical plates** move the beam **up and down**, while **horizontal plates** move it **right and left**. The two sets of plates are physically separated to prevent interaction of the field. The position of the spot at any instant is a resultant of potentials on the two set of plates at that instant.

The viewing **screen** is created by **phosphor coating** inside front of the tube. When electron beam strikes the screen of CRT with considerable energy, the phosphor absorbs the kinetic energy of bombarding electrons and reemits energy at a lower frequency range in visible spectrum. Thus a spot of light is produced in outside front of the screen. In addition to **light, heat** as well as **secondary electrons** of low energy is generating. **Aquadag coating** of graphite material is cover the inside surface of CRT nearly up the screen to remove these secondary electrons.

The property of some crystalline materials such as **phosphor or zinc oxide** to emit light when stimulates by radiation is called **fluorescence**.

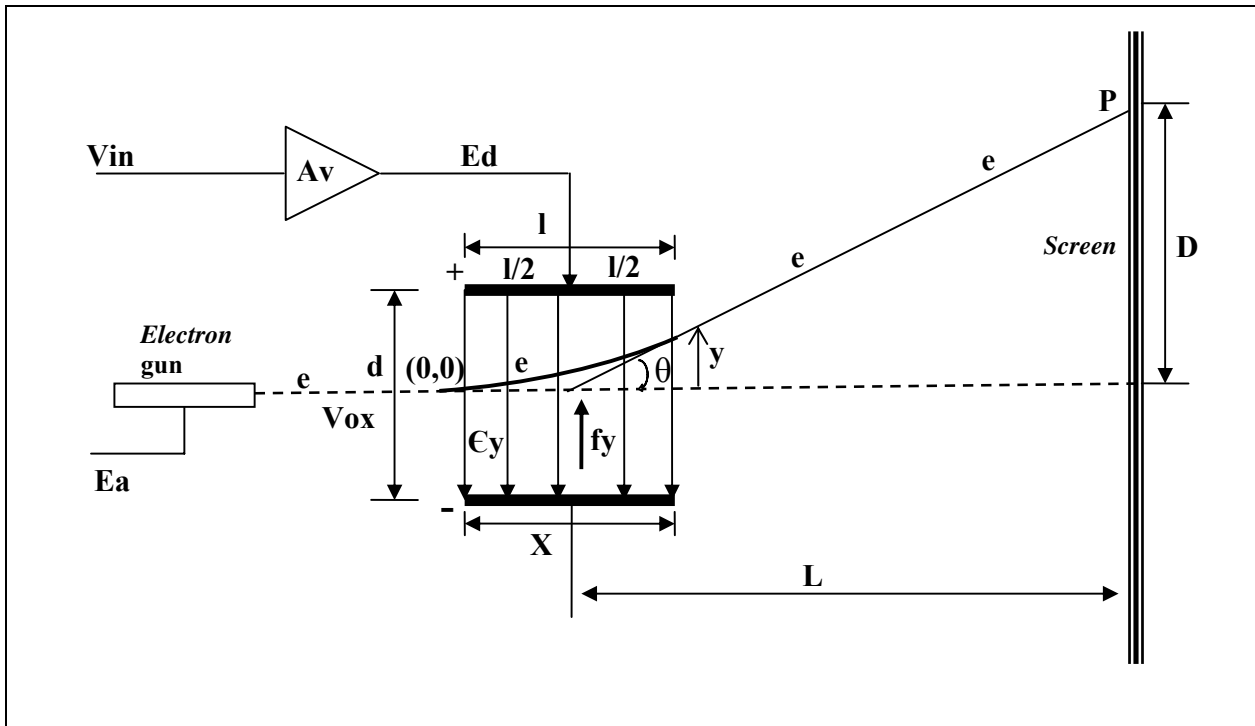
Phosphorescence refers to the property of material to continue **light emission** even after the source of excitation is **cut off**.

Persistence is the length of **time** that the intensity of spot is taken to **decrease** to 10% of its original brightness.

Finally, the working parts of a CRT are enclosed in a high vacuum glass envelope to permit the electron beam moves freely from one end to other with out collision.

Graticules is a set of horizontal and vertical lines permanently scribed on CRT face to allow easily measured the waveform values.

Electrostatic Deflection Equations:



V_{in} where V_{in} : input voltage to channel A or B of CRO

\Downarrow
 $E_d = V_{in} \cdot A_v$

E_d : deflection voltage (potential)
 $E_x = E_z = 0$

\Downarrow
 $\epsilon_y = \frac{-E_d}{d}$ (1) ϵ_y : electrical field in Y direction

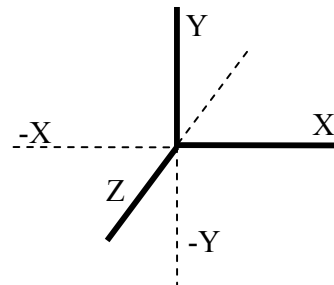
\Downarrow
 $f_y = -e \cdot \epsilon_y$ $\epsilon_x = \epsilon_z = 0$
 \Downarrow f_y : force generate by electrical field effect
 e : electron charge (1.6×10^{-19} C)

\Downarrow
 $a_y = \frac{f_y}{m_e}$ a_y : acceleration in Y direction ,
 \Downarrow $f_x = f_z = 0$
 $a_x = a_z = 0$

\Downarrow m_e : electron mass (9.1×10^{-31} Kg) $V_x = V_{ox} = \text{constant}$ $V_z = 0$
 $V_y = V_{oy} + a_y t$ Since $V_{oy} = 0$ V_y : velocity in Y direction at any time

\Downarrow
 $V_y = a_y t = \frac{f_y}{m_e} t = \frac{-e}{m_e} \epsilon_y t$ V_{oy} : initial velocity in Y direction

\Downarrow
 $Y = Y_o + V_{oy} t + \frac{1}{2} a_y t^2$ Since $Y_o = 0$ $V_{oy} = 0$ Y : distance in Y direction



$$Y = \frac{1}{2} a_y t^2 = \frac{-1}{2} \frac{e}{m_e} \epsilon_y t^2$$

Yo: initial distance in Y direction

$$Y = \frac{-1}{2} \frac{e}{m_e} \epsilon_y t^2 \dots\dots\dots (2)$$

Relation of Y with time

$$V_x = V_{ox} + a_x t \quad \text{Since} \quad a_x = 0$$

$$V_x = V_{ox}$$

Vx: velocity in X direction

↓

$$X = X_o + V_{ox} t + \frac{1}{2} a_x t^2 \quad \text{Since} \quad X_o = 0 \quad \frac{1}{2} a_x t^2 = 0$$

$$X = V_{ox} t \dots\dots\dots (3)$$

Relation of X with time

$$t = \frac{X}{V_{ox}} \dots\dots\dots (4)$$

Substitute equ. (4) into equ.(2) give

$$Y = \frac{-1}{2} \frac{e}{m_e} \epsilon_y \frac{X^2}{V_{ox}^2} \dots\dots\dots (5) \text{ The parabolic equation of electron beam}$$

$$\frac{1}{2} m V_{ox}^2 = e E a \quad \text{where (Ea) is the acceleration voltage (potential)}$$

$$V_{ox} = \sqrt{\frac{2eEa}{m}} \dots\dots\dots (6)$$

By substituting eqs.(6) & (1) into equ.(5) we get

$$Y = \left(\frac{1}{4d} \frac{Ed}{Ea} \right) \cdot X^2 \dots\dots\dots (7) \quad \Leftarrow \text{ Relation of Y with X}$$

When the electrons leaves the region of deflecting plates, the deflecting force no longer exist, and the electrons travels in a straight line toward point P. The slope of parabolic curve at distance (x=l) is:

$$\tan \theta = \frac{dy}{dx} = \frac{-el}{m V_{ox}^2} \epsilon_y$$

Or

$$\tan \theta = \left(\frac{1}{2d} \frac{Ed}{Ea} \right) l \dots\dots\dots (8)$$

The deflection on the screen (D) is

$$D = L \tan \theta \dots\dots\dots (9)$$

Substitute equ.(9) into (8) give

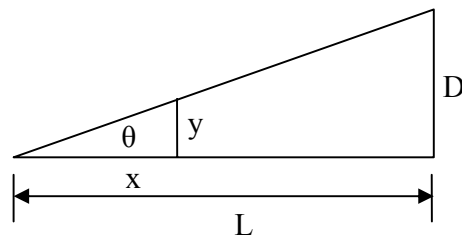
$$D = \frac{L E d}{2 d E a} \dots\dots\dots (10)$$

♣ The deflection sensitivity (S) of CRT is:

$$S = \frac{D}{Ed} \dots\dots\dots (11)$$

♣ The deflection factor (G) of CRT is:

$$G = \frac{1}{S} = \frac{Ed}{D} = \frac{2dEa}{lL} \dots\dots\dots (12)$$



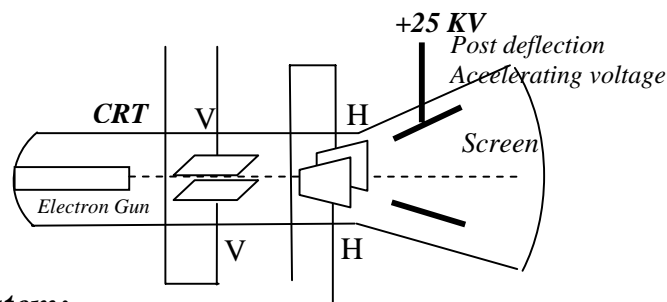
By similarity

$$\frac{y}{x} = \frac{D}{L}$$

Post Deflection Acceleration:

The amount of light given off by the phosphor depends on the amount of energy that is transferred to the phosphor by the electron beam. For **fast oscilloscope** (of high frequency response greater than 100MHz), the **velocity** of electron beam must be **great** to respond to fast occurring events; otherwise, the light output will be drop off. This is done by increasing the acceleration potential but it will be difficult to deflected the fast electron beam by the deflection plates because this would required a higher deflection voltage and a higher deflection current to charge the capacitance of the plates.

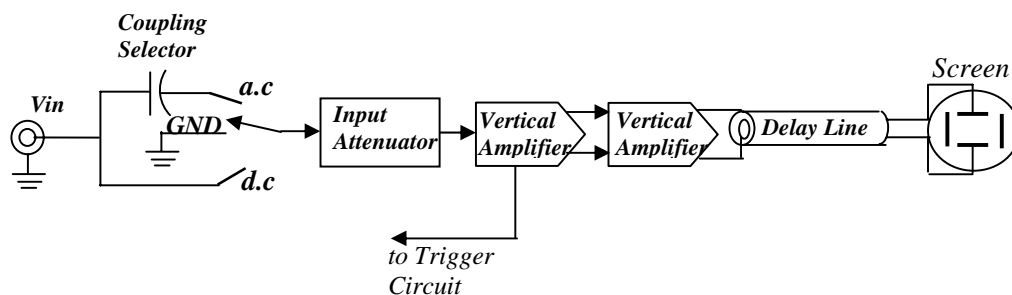
Modern CRTs use a two step acceleration to eliminate this problem. **First**, the electron beam is **accelerated** to a relatively **low velocity** through a potential of a few thousand volts. The beam is then deflected and **after deflection is further accelerated to the desired final velocity**. The **deflection sensitivity** of the CRT **depends** on the **acceleration voltage** before the deflection plates, which is usually regulated and does not depends on the post acceleration voltage after the deflection plates.



Vertical deflection system:

The vertical deflection system provides an amplified signal of the proper level to derive the vertical deflection plates with out introducing any appreciable distortion into the system. This system is consists of the following elements:

- 1- Input coupling selector.
- 2- Input attenuator.
- 3- Preamplifier.
- 4- Main vertical amplifier.
- 5- Delay line.



Vertical Deflection System

1- Input Coupling Selector:

Its purpose is to allow the oscilloscope more flexibility in the display of certain types of signals. For example, an input signal may be a d.c signal, an a.c signal, or a.c component superimposed on a d.c component. There are three positions switch in the coupling selector (d.c, a.c, and GND). If an *a.c* position is chosen, the capacitor appears as an open circuit to the d.c components and hence block them from entering. While the GND position ground the internal circuitry of the amplifier to remove any stored charge and recenter the electron beam.

2- 4- Input Attenuators And Amplifiers:

The combine operation of the attenuator, preamplifier and main amplifier together make up the amplifying portion of the system.

The function of the attenuator is to reduce the amplitude of the input signal by a selected factor and verse varies amplifier function.

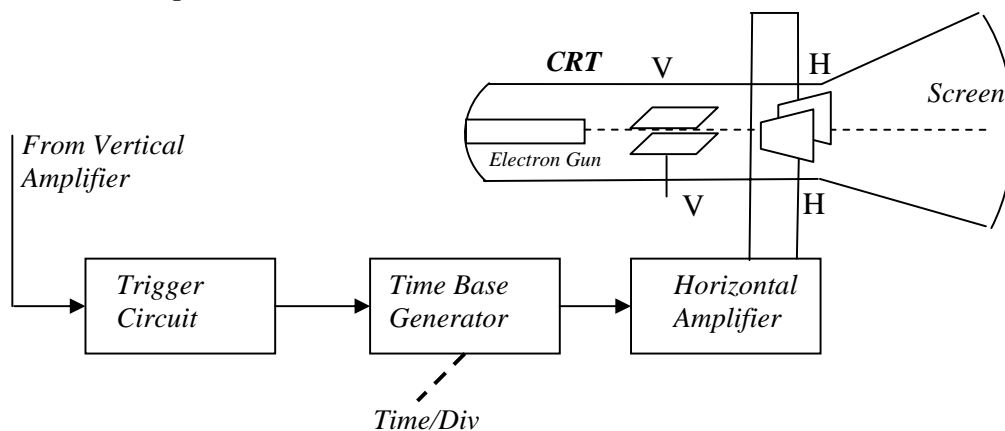
5- Delay Line:

Since part of the input signal is picked off and fed to the horizontal deflection system to initiate a sweep waveform that is synchronized with the leading edge of the input signal. So the purpose of delay is to delay the vertical amplified signal from reaching the vertical plates until the horizontal signal reach the horizontal plates to begin together at the same time on CRT screen.

Horizontal Deflection System:

The horizontal deflection system of OSC consist of :

- 1- Trigger circuit.
- 2- Time base generator.
- 3- Horizontal amplifier.



Horizontal Deflection System

Trigger and Time Base Generator:

The most common application of an oscilloscope is the display of voltage variation versus time. To generate this type of display a saw tooth waveform is applied to horizontal plates. The electron beam being bent towards the more positive plate and deflected the luminous spot from left to right of the screen at constant velocity whilst the return or fly back is at a speed in excess of the maximum writing speed and hence invisible. The saw tooth or time base signal must be repetitively applied to the horizontal plates so that; the beam can retrace the same path rapidly enough to make the moving spot of light appear to be a solid line.

To synchronous the time base signal applied to (X-plates) with input voltage to be measured which applied to vertical or (Y-plates) a triggering circuit is used. This circuit is sensitive to the level of voltage applied to it, so that when a predetermined level of voltage is reached a pulse is passed from the trigger circuit to initiate one sweep of the time base. In a practical oscilloscope the time base will be adjustable from the front panel control of scope.

Horizontal Amplifier:

The horizontal amplifier is used to amplify the sweep waveform to the required level of horizontal plates operation.