31 Mesh-current and nodal analysis

At the end of this chapter you should be able to:

- solve d.c. and a.c. networks using mesh-current analysis
- solve d.c. and a.c. networks using nodal analysis

31.1 Mesh-current analysis

Mesh-current analysis is merely an extension of the use of Kirchhoff’s laws, explained in Chapter 30. Figure 31.1 shows a network whose circulating currents $I_1$, $I_2$ and $I_3$ have been assigned to closed loops in the circuit rather than to branches. Currents $I_1$, $I_2$ and $I_3$ are called mesh-currents or loop-currents.

![Figure 31.1](mywbut.com)

In mesh-current analysis the loop-currents are all arranged to circulate in the same direction (in Figure 31.1, shown as clockwise direction). Kirchhoff’s second law is applied to each of the loops in turn, which in the circuit of Figure 31.1 produces three equations in three unknowns which may be solved for $I_1$, $I_2$ and $I_3$. The three equations produced from Figure 31.1 are:

$$I_1(Z_1 + Z_2) - I_2Z_2 = E_1$$
$$I_2(Z_2 + Z_3 + Z_4) - I_1Z_2 - I_3Z_4 = 0$$
$$I_3(Z_4 + Z_5) - I_2Z_4 = -E_2$$

The branch currents are determined by taking the phasor sum of the mesh currents common to that branch. For example, the current flowing
in impedance $Z_2$ of Figure 31.1 is given by $(I_1 - I_2)$ phasorially. The method of mesh-current analysis, called Maxwell's theorem, is demonstrated in the following problems.

Problem 1. Use mesh-current analysis to determine the current flowing in (a) the 5 Ω resistance, and (b) the 1 Ω resistance of the d.c. circuit shown in Figure 31.2.

![Figure 31.2](image_url)

The mesh currents $I_1$, $I_2$ and $I_3$ are shown in Figure 31.2. Using Kirchhoff’s voltage law:

For loop 1, \[(3 + 5)I_1 - 5I_2 = 4\] (1)

For loop 2, \[(4 + 1 + 6 + 5)I_2 - (5)I_1 - (1)I_3 = 0\] (2)

For loop 3, \[(1 + 8)I_3 - (1)I_2 = -5\] (3)

Thus

\[8I_1 - 5I_2 - 4 = 0\] (1′)

\[-5I_1 + 16I_2 - I_3 = 0\] (2′)

\[-I_2 + 9I_3 + 5 = 0\] (3′)

Using determinants,

\[
\begin{vmatrix}
-5 & 0 & -4 \\
16 & -1 & 0 \\
-1 & 9 & 5 \\
\end{vmatrix}
= \begin{vmatrix}
8 & 0 & -4 \\
-5 & -1 & 0 \\
0 & 9 & 5 \\
\end{vmatrix}
= \begin{vmatrix}
8 & -5 & -4 \\
-5 & 16 & 0 \\
0 & -1 & 5 \\
\end{vmatrix}
= -1
\]

\[
\begin{vmatrix}
8 & -5 & 0 \\
-5 & 16 & -1 \\
0 & -1 & 9 \\
\end{vmatrix}
= 1
\]
Hence $I_1 = \frac{547}{919} = 0.595 \text{ A}$, $I_2 = \frac{140}{919} = 0.152 \text{ A}$, and $I_3 = \frac{-495}{919} = -0.539 \text{ A}$.

Thus current in the 5 $\Omega$ resistance $= I_1 - I_2 = 0.595 - 0.152 = 0.44 \text{ A}$,
and current in the 1 $\Omega$ resistance $= I_2 - I_3 = 0.152 - (-0.539) = 0.69 \text{ A}$

Problem 2. For the a.c. network shown in Figure 31.3 determine, using mesh-current analysis, (a) the mesh currents $I_1$ and $I_2$ (b) the current flowing in the capacitor, and (c) the active power delivered by the 100$\times$0$^\circ$ V voltage source.

(a) For the first loop $(5 - j4)I_1 - (-j4)I_2 = 100\times0^\circ$  (1)

For the second loop $(4 + j3 - j4)I_2 - (-j4I_1) = 0$  (2)

Rewriting equations (1) and (2) gives:

$(5 - j4)I_1 + j4I_2 - 100 = 0$  (1')

$j4I_1 + (4 - j)I_2 + 0 = 0$  (2')
Thus, using determinants,
\[
\begin{vmatrix}
  j4 & -100 \\
  (4 - j) & 0 \\
\end{vmatrix}
\begin{vmatrix}
  (5 - j4) & -100 \\
  j4 & 0 \\
\end{vmatrix}
\begin{vmatrix}
  (5 - j4) & j4 \\
  j4 & (4 - j) \\
\end{vmatrix}
\]
\[
\begin{align*}
I_1 = -I_2 & \Rightarrow \frac{(400 - j100)}{(32 - j21)} = \frac{1}{(32 - j21)} \\
& \Rightarrow \frac{412.31 - 14.04^\circ}{38.28 - 33.27^\circ} \\
& = 10.77 - 19.23^\circ \text{ A, correct to one decimal place}
\end{align*}
\]
\[
I_2 = \frac{400 - 90^\circ}{38.28 - 33.27^\circ} = 10.45 - 56.73^\circ \text{ A, correct to one decimal place}
\]

(b) Current flowing in capacitor = \( I_1 - I_2 \)
\[
= 10.77 - 19.23^\circ - 10.45 - 56.73^\circ \\
= 4.44 + j12.28 = 13.1 - 70.12^\circ \text{ A, i.e., the current in the capacitor is 13.1 A}
\]

(c) Source power \( P = VI \cos \phi = (100)(10.77) \cos 19.23^\circ \)
\[
= 1016.9 \text{ W} = 1020 \text{ W, correct to three significant figures.}
\]

(\text{Check: power in 5 } \Omega \text{ resistor } = I_1^2(5) = (10.77)^2(5) = 579.97 \text{ W})

and power in 4 \( \Omega \) resistor = \( I_2^2(4) = (10.45)^2(4) = 436.81 \text{ W} \)

Thus total power dissipated = 579.97 + 436.81
\[
= 1016.8 \text{ W } = 1020 \text{ W, correct to three significant figures.}
\]

Problem 3. A balanced star-connected 3-phase load is shown in Figure 31.4. Determine the value of the line currents \( I_R, I_Y \) and \( I_B \) using mesh-current analysis.
Two mesh currents $I_1$ and $I_2$ are chosen as shown in Figure 31.4.

From loop 1, $I_1(3 + j4) + I_1(3 + j4) - I_2(3 + j4) = 415\angle 120^\circ$

\[
i.e., \quad (6 + j8)I_1 - (3 + j4)I_2 - 415\angle 120^\circ = 0 \quad (1)
\]

From loop 2, $I_2(3 + j4) - I_1(3 + j4) + I_2(3 + j4) = 415\angle 0^\circ$

\[
i.e., \quad - (3 + j4)I_1 + (6 + j8)I_2 - 415\angle 0^\circ = 0 \quad (2)
\]

Solving equations (1) and (2) using determinants gives:

\[
\begin{vmatrix}
-3 + j4 & -415\angle 120^\circ \\
6 + j8 & -415\angle 0^\circ \\
\end{vmatrix}
\text{= } \frac{-3 + j4}{-415\angle 120^\circ - 415\angle 0^\circ}
\]

\[
\begin{vmatrix}
6 + j8 & -415\angle 120^\circ \\
-3 + j4 & -415\angle 0^\circ \\
\end{vmatrix}
\]

\[
\begin{vmatrix}
6 + j8 & -3 + j4 \\
-3 + j4 & 6 + j8 \\
\end{vmatrix}
\]

\[
\begin{vmatrix}
I_1 \\
2075\angle 53.13^\circ + 4150\angle 173.13^\circ \\
\end{vmatrix}
\text{= } \frac{-I_2}{-4150\angle 53.13^\circ - 2075\angle 173.13^\circ}
\]

\[
\begin{vmatrix}
I_1 \\
100\angle 106.26^\circ - 25\angle 106.26^\circ \\
\end{vmatrix}
\]

\[
\begin{vmatrix}
I_2 \\
3594\angle 143.13^\circ \\
\end{vmatrix}
\text{= } \frac{I_2}{3594\angle 83.13^\circ}
\]

\[
\begin{vmatrix}
I_2 \\
75\angle 106.26^\circ \\
\end{vmatrix}
\]

Hence $I_1 = \frac{3594\angle 143.13^\circ}{75\angle 106.26^\circ} = 47.9\angle 36.87^\circ$ A

and $I_2 = \frac{3594\angle 83.13^\circ}{75\angle 106.26^\circ} = 47.9\angle -23.13^\circ$ A

Thus line current $I_B = I_1 = 47.9\angle 36.87^\circ$ A

\[
I_B = -I_2 = -(47.9\angle -23.23^\circ) A
\]

\[
= 47.9\angle 156.87^\circ A
\]

and

\[
I_Y = I_2 - I_1 = 47.9\angle -23.13^\circ - 47.9\angle 36.87^\circ
\]

\[
= 47.9\angle -83.13^\circ A
\]

*Further problems on mesh-current analysis may be found in Section 31.3, problems 1 to 9, page 559.*
31.2 Nodal analysis

A node of a network is defined as a point where two or more branches are joined. If three or more branches join at a node, then that node is called a principal node or junction. In Figure 31.5, points 1, 2, 3, 4 and 5 are nodes, and points 1, 2 and 3 are principal nodes.

A node voltage is the voltage of a particular node with respect to a node called the reference node. If in Figure 31.5, for example, node 3 is chosen as the reference node then \( V_{13} \) is assumed to mean the voltage at node 1 with respect to node 3 (as distinct from \( V_{31} \)). Similarly, \( V_{23} \) would be assumed to mean the voltage at node 2 with respect to node 3, and so on. However, since the node voltage is always determined with respect to a particular chosen reference node, the notation \( V_1 \) for \( V_{13} \) and \( V_2 \) for \( V_{23} \) would always be used in this instance.

The object of nodal analysis is to determine the values of voltages at all the principal nodes with respect to the reference node, e.g., to find voltages \( V_1 \) and \( V_2 \) in Figure 31.5. When such voltages are determined, the currents flowing in each branch can be found.

Kirchhoff’s current law is applied to nodes 1 and 2 in turn in Figure 31.5 and two equations in unknowns \( V_1 \) and \( V_2 \) are obtained which may be simultaneously solved using determinants.

![Figure 31.5](image1)

![Figure 31.6](image2)

The branches leading to node 1 are shown separately in Figure 31.6. Let us assume that all branch currents are leaving the node as shown. Since the sum of currents at a junction is zero,

\[
\frac{V_1 - V_x}{Z_A} + \frac{V_1}{Z_D} + \frac{V_1 - V_2}{Z_B} = 0
\]

(1)

Similarly, for node 2, assuming all branch currents are leaving the node as shown in Figure 31.7,

\[
\frac{V_2 - V_1}{Z_B} + \frac{V_2}{Z_E} + \frac{V_2 + V_Y}{Z_C} = 0
\]

(2)

In equations (1) and (2), the currents are all assumed to be leaving the node. In fact, any selection in the direction of the branch currents may be made — the resulting equations will be identical. (For example, if for node 1 the current flowing in \( Z_B \) is considered as flowing towards node 1 instead of away, then the equation for node 1 becomes...
\[ \frac{V_1 - V_x}{Z_A} + \frac{V_1}{Z_D} = \frac{V_2 - V_1}{Z_B} \]

which if rearranged is seen to be exactly the same as equation (1).

Rearranging equations (1) and (2) gives:

\[ \left(\frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_D}\right) V_1 - \left(\frac{1}{Z_B}\right) V_2 - \left(\frac{1}{Z_A}\right) V_x = 0 \quad (3) \]
\[ -\left(\frac{1}{Z_B}\right) V_1 + \left(\frac{1}{Z_B} + \frac{1}{Z_C} + \frac{1}{Z_E}\right) V_2 + \left(\frac{1}{Z_C}\right) V_Y = 0 \quad (4) \]

Equations (3) and (4) may be rewritten in terms of admittances (where admittance \( Y = 1/Z \)):

\[ (Y_A + Y_B + Y_D)V_1 - Y_BV_2 - Y_AV_x = 0 \quad (5) \]
\[ -Y_BV_1 + (Y_B + Y_C + Y_E)V_2 + Y_CY_Y = 0 \quad (6) \]

Equations (5) and (6) may be solved for \( V_1 \) and \( V_2 \) by using determinants.

Thus

\[
\begin{vmatrix}
V_1 & -V_2 \\
-Y_B & -Y_A \\
(Y_B + Y_C + Y_E) & Y_C
\end{vmatrix} =
\begin{vmatrix}
(Y_A + Y_B + Y_D) & -Y_A \\
-Y_B & Y_C
\end{vmatrix}
\]

\[
= \frac{1}{(Y_A + Y_B + Y_D)(Y_B + Y_C + Y_E) - Y_B(Y_A + Y_B + Y_D)}
\]

Current equations, and hence voltage equations, may be written at each principal node of a network with the exception of a reference node. The number of equations necessary to produce a solution for a circuit is, in fact, always one less than the number of principal nodes.

Whether mesh-current analysis or nodal analysis is used to determine currents in circuits depends on the number of loops and nodes the circuit contains. Basically, the method that requires the least number of equations is used. The method of nodal analysis is demonstrated in the following problems.

**Problem 4.** For the network shown in Figure 31.8, determine the voltage \( V_{AB} \), by using nodal analysis.

![Diagram of Figure 31.8](mywbut.com)
Figure 31.8 contains two principal nodes (at 1 and B) and thus only one nodal equation is required. B is taken as the reference node and the equation for node 1 is obtained as follows. Applying Kirchhoff’s current law to node 1 gives:

\[ I_X + I_Y = I \]

i.e.,

\[ \frac{V_1}{16} + \frac{V_1}{(4 + j3)} = 20° \]

Thus

\[ V_1 \left( \frac{1}{16} + \frac{1}{4 + j3} \right) = 20 \]

\[ V_1 \left( 0.0625 + \frac{4 - j3}{4^2 + 3^2} \right) = 20 \]

\[ V_1(0.0625 + 0.16 - j0.12) = 20 \]

\[ V_1(0.2225 - j0.12) = 20 \]

from which,

\[ V_1 = \frac{20}{0.2225 - j0.12} = \frac{20}{0.2528° - 28.34°} \]

i.e., voltage \( V_1 = 79.1° 28.34° \) V

The current through the (4 + j3)Ω branch, \( I_Y = V_1/(4 + j3) \)

Hence the voltage drop between points A and B, i.e., across the 4 Ω resistance, is given by:

\[ V_{AB} = (I_Y)(4) = \frac{V_1(4)}{(4 + j3)} = \frac{79.1° 28.34°}{5° 36.87°}(4) = 63.3° - 8.53° \) V

Problem 5. Determine the value of voltage \( V_{XY} \) shown in the circuit of Figure 31.9.

The circuit contains no principal nodes. However, if point Y is chosen as the reference node then an equation may be written for node X assuming that current leaves point X by both branches.

Thus

\[ \frac{V_X - 8° 0°}{(5 + 4)} + \frac{V_x - 8° 90°}{(3 + j6)} = 0 \]

from which,

\[ V_X \left( \frac{1}{9} + \frac{1}{3 + j6} \right) = \frac{8}{9} + \frac{j8}{3 + j6} \]

\[ V_X \left( \frac{1}{9} + \frac{3 - j6}{3^2 + 6^2} \right) = \frac{8}{9} + \frac{j8(3 - j6)}{3^2 + 6^2} \]
Since point \( Y \) is the reference node, voltage \( V_X = V_{XY} = \frac{2.027\angle15.25^\circ}{0.2222\angle-36.86^\circ} = 9.12\angle52.11^\circ \text{ V} \)

Problem 6. Use nodal analysis to determine the current flowing in each branch of the network shown in Figure 31.10.

This is the same problem as problem 1 of Chapter 30, page 536, which was solved using Kirchhoff’s laws. A comparison of methods can be made.

There are only two principal nodes in Figure 31.10 so only one nodal equation is required. Node 2 is taken as the reference node.

The equation at node 1 is \( I_1 + I_2 + I_3 = 0 \)

i.e., \[ \frac{V_1 - 100\angle0^\circ}{25} + \frac{V_1}{20} + \frac{V_1 - 50\angle90^\circ}{10} = 0 \]

i.e., \( \left( \frac{1}{25} + \frac{1}{20} + \frac{1}{10} \right)V_1 = \frac{100\angle0^\circ}{25} - \frac{50\angle90^\circ}{10} = 0 \)

\[ 0.19V_1 = 4 + j5 \]

Thus the voltage at node 1, \( V_1 = \frac{4 + j5}{0.19} = 33.70\angle51.34^\circ \text{ V} \)

or \( (21.05 + j26.32)\text{ V} \)

Hence the current in the 25 \( \Omega \) resistance,

\[ I_1 = \frac{V_1 - 100\angle0^\circ}{25} = \frac{21.05 + j26.32 - 100}{25} \]

\[ = \frac{-78.95 + j26.32}{25} \]

\[ = 3.33\angle161.56^\circ \text{ A} \text{ flowing away from node 1} \]

(or \( 3.33\angle(161.56^\circ - 180^\circ) \text{A} = 3.33\angle-18.44^\circ \text{ A} \text{ flowing toward node 1} \))

The current in the 20 \( \Omega \) resistance,

\[ I_2 = \frac{V_1}{20} = \frac{33.70\angle51.34^\circ}{20} = 1.69\angle51.34^\circ \text{ A} \text{ flowing from node 1 to node 2} \]
The current in the 10 Ω resistor,

\[ I_3 = \frac{V_1 - 50 \angle 90^\circ}{10} = \frac{21.05 + j26.32 - j50}{10} = \frac{21.05 - j23.68}{10} \]

\[ = 3.17 \angle -48.36^\circ \text{ A away from node 1} \]

(or \(3.17 \angle (-48.36^\circ - 180^\circ) = 3.17 \angle -228.36^\circ \text{ A} = 3.17 \angle 131.64^\circ \text{ A toward node 1} \))

Problem 7. In the network of Figure 31.11 use nodal analysis to determine (a) the voltage at nodes 1 and 2, (b) the current in the \(j4\) Ω inductance, (c) the current in the 5 Ω resistance, and (d) the magnitude of the active power dissipated in the 2.5 Ω resistance.

![Figure 31.11](image)

(a) At node 1, \( \frac{V_1 - 25 \angle 0^\circ}{2} + \frac{V_1}{-j4} + \frac{V_1 - V_2}{5} = 0 \)

Rearranging gives:

\[ \left( \frac{1}{2} + \frac{1}{-j4} + \frac{1}{5} \right) V_1 - \left( \frac{1}{5} \right) V_2 - \frac{25 \angle 0^\circ}{2} = 0 \]

i.e., \((0.7 + j0.25)V_1 - 0.2V_2 - 12.5 = 0\) \hspace{1cm} (1)

At node 2, \( \frac{V_2 - 25 \angle 90^\circ}{2.5} + \frac{V_2}{j4} + \frac{V_2 - V_1}{5} = 0 \)

Rearranging gives:

\[ -\left( \frac{1}{5} \right) V_1 + \left( \frac{1}{2.5} + \frac{1}{j4} + \frac{1}{5} \right) V_2 - \frac{25 \angle 90^\circ}{2.5} = 0 \]

i.e., \(-0.2V_1 + (0.6 - j0.25)V_2 - j10 = 0\) \hspace{1cm} (2)

Thus two simultaneous equations have been formed with two unknowns, \(V_1\) and \(V_2\). Using determinants, if \((0.7 + j0.25)V_1 - 0.2V_2 - 12.5 = 0\) \hspace{1cm} (1)
and \(-0.2V_1 + (0.6 - j0.25)V_2 - j10 = 0\) \hspace{1cm} (2)

\[
\begin{vmatrix}
-0.2 & -12.5 \\
(0.6 - j0.25) & -j10
\end{vmatrix} = \begin{vmatrix}
-V_2 \\
(0.7 + j0.25) & -12.5 \\
-0.2 & -j10
\end{vmatrix} = 1
= \begin{vmatrix}
0.7 + j0.25 & -0.2 \\
-0.2 & (0.6 - j0.25)
\end{vmatrix}
\]

i.e.,
\[
\frac{V_1}{(j2 + 7.5 - j3.125)} = \frac{-V_2}{(-j7 + 2.5 - 2.5)} = \frac{1}{(0.42 - j0.175 + j0.15 + 0.0625 - 0.04)}
\]

and
\[
\frac{V_1}{7.584\angle -8.53^\circ} = \frac{-V_2}{-7.90^\circ} = \frac{1}{0.443\angle -3.23^\circ}
\]

Thus \textit{voltage}, \(V_1 = \frac{7.584\angle -8.53^\circ}{0.443\angle -3.23^\circ} = 17.12\angle -5.30^\circ \text{ V}\)

\[= 17.1\angle -5.3^\circ \text{ V, correct to one decimal place,}\]

and \textit{voltage}, \(V_2 = \frac{7\angle90^\circ}{0.443\angle -3.23^\circ} = 15.80\angle93.23^\circ \text{ V}\)

\[= 15.8\angle93.2^\circ \text{ V, correct to one decimal place.}\]

(b) The current in the \(j4 \Omega\) inductance is given by:
\[
\frac{V_2}{j4} = \frac{15.80\angle93.23^\circ}{4\angle90^\circ} = 3.95\angle3.23^\circ \text{ A flowing away from node 2}\]

(c) The current in the 5 \(\Omega\) resistance is given by:
\[
I_5 = \frac{V_1 - V_2}{5} = \frac{17.12\angle -5.30^\circ - 15.80\angle93.23^\circ}{5}
\]
i.e.,
\[
I_5 = \frac{(17.05 - j1.58) - (-0.89 + j15.77)}{5}
= \frac{17.94 - j17.35}{5} = \frac{24.96\angle -44.04^\circ}{5}
= 4.99\angle -44.04^\circ \text{ A flowing from node 1 to node 2}\]

(d) The active power dissipated in the 2.5 \(\Omega\) resistor is given by
\[
P_{2.5} = (I_{2.5})^2(2.5) = \left(\frac{V_2 - 25\angle90^\circ}{2.5}\right)^2(2.5)
= \frac{(0.89 + j15.77 - j25)^2}{2.5} = \frac{9.273\angle -95.51^\circ}{2.5}\]
Thus the magnitude of the active power dissipated in the 2.5 $\Omega$ resistance is 34.4 W.

Problem 8. In the network shown in Figure 31.12 determine the voltage $V_{XY}$ using nodal analysis.

Node 3 is taken as the reference node.

At node 1, \[25^\circ = \frac{V_1}{4+j3} + \frac{V_1 - V_2}{5}\]
i.e., \[(4-j3/25+1/5)V_1-\frac{1}{5}V_2-25=0\]
or \[(0.379-18.43^\circ)V_1-0.2V_2-25=0\]  \hspace{1cm} (1)

At node 2, \[\frac{V_2}{j10} + \frac{V_2}{j20} + \frac{V_2 - V_1}{5} = 0\]
i.e., \[-0.2V_1 + \left(\frac{1}{j10} + \frac{1}{j20} + \frac{1}{5}\right)V_2 = 0\]
or \[-0.2V_1 + (-j0.1 - j0.05 + 0.2)V_2 = 0\]
i.e., \[-0.2V_1 + (0.25-36.87^\circ)V_2 + 0 = 0\]  \hspace{1cm} (2)

Simultaneous equations (1) and (2) may be solved for $V_1$ and $V_2$ by using determinants. Thus,
Problem 9. Use nodal analysis to determine the voltages at nodes 2 and 3 in Figure 31.13 and hence determine the current flowing in the 2 Ω resistor and the power dissipated in the 3 Ω resistor.

This is the same problem as Problem 2 of Chapter 30, page 537, which was solved using Kirchhoff’s laws.

In Figure 31.13, the reference node is shown at point A.

At node 1,
\[ \frac{V_1 - V_2}{1} + \frac{V_1}{6} + \frac{V_1 - 8 - V_3}{5} = 0 \]
i.e.,
\[ 1.367V_1 - V_2 - 0.2V_3 - 1.6 = 0 \]  \hspace{1cm} (1)

At node 2,
\[ \frac{V_2}{2} + \frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{3} = 0 \]
i.e., 
\[-V_1 + 1.833V_2 - 0.333V_3 + 0 = 0 \tag{2}\]

At node 3,
\[\frac{V_3}{4} + \frac{V_3 - V_2}{3} + \frac{V_3 + 8 - V_1}{5} = 0\]

i.e.,
\[-0.2V_1 - 0.333V_2 + 0.783V_3 + 1.6 = 0 \tag{3}\]

Equations (1) to (3) can be solved for $V_1$, $V_2$ and $V_3$ by using determinants. Hence

\[
\begin{vmatrix}
-1 & -0.2 & -1.6 \\
1.833 & -0.333 & 0 \\
-0.333 & 0.783 & 1.6
\end{vmatrix}
= \begin{vmatrix}
1.367 & -0.2 & -1.6 \\
-1 & -0.333 & 0 \\
-0.2 & 0.783 & 1.6
\end{vmatrix}
= \begin{vmatrix}
1.367 & -1 & -0.2 \\
-1 & 1.833 & -0.333 \\
-0.2 & -0.333 & 0.783
\end{vmatrix}

Solving for $V_2$ gives:
\[-V_2 = \frac{-1.6(-0.8496) + 1.6(-0.6552)}{1.367(1.3244) + 1(-0.8496) - 0.2(0.6996)} = \frac{-1}{0.31104} = \frac{-0.31104}{0.82093} = 0.3789 \text{ V}\]

Thus the current in the 2 Ω resistor $= \frac{V_2}{2} = \frac{0.3789}{2} = 0.19$ A, flowing from node 2 to node A.

Solving for $V_3$ gives:
\[-V_3 = \frac{-1.6(0.6996) + 1.6(1.5057)}{0.82093} = \frac{-1}{1.2898} = \frac{-1.2898}{0.82093} = -1.571 \text{ V}\]

hence $V_3 = \frac{-1.2898}{0.82093} = -1.571 \text{ V}\]

Power in the 3 Ω resistor $= (I_3)^2(3) = \left(\frac{V_2 - V_3}{3}\right)^2 = \frac{(0.3789 - (-1.571))^2}{3} = 1.27 \text{ W}$

Further problems on nodal analysis may be found in Section 31.3 following, problems 10 to 15, page 560.
31.3 Further problems on mesh-current and nodal analysis

Mesh-current analysis

1. Repeat problems 1 to 10, page 542, of Chapter 30 using mesh-current analysis.

2. For the network shown in Figure 31.14, use mesh-current analysis to determine the value of current $I$ and the active power output of the voltage source. 
\[ 6.96\angle-49.94^\circ \, \text{A} ; 644 \, \text{W} \]

3. Use mesh-current analysis to determine currents $I_1, I_2$ and $I_3$ for the network shown in Figure 31.15.
\[
I_1 = 8.73\angle-1.37^\circ \, \text{A}, \\
I_2 = 7.02\angle17.25^\circ \, \text{A}, \\
I_3 = 3.05\angle-48.67^\circ \, \text{A}
\]

4. For the network shown in Figure 31.16, determine the current flowing in the $(4 + j3)\Omega$ impedance.

5. For the network shown in Figure 31.17, use mesh-current analysis to determine (a) the current in the capacitor, $I_C$, (b) the current in the inductance, $I_L$, (c) the p.d. across the $4 \Omega$ resistance, and (d) the total active circuit power.
\[(a) \, 14.5 \, \text{A} \quad (b) \, 11.5 \, \text{A} \quad (c) \, 71.8 \, \text{V} \quad (d) \, 2499 \, \text{W} \]

6. Determine the value of the currents $I_R, I_Y$ and $I_B$ in the network shown in Figure 31.18 by using mesh-current analysis.
\[
I_R = 7.84\angle71.19^\circ \, \text{A} ; I_Y = 9.04\angle-37.50^\circ \, \text{A} ; \\
I_B = 9.89\angle-168.81^\circ \, \text{A}
\]

7. In the network of Figure 31.19, use mesh-current analysis to determine (a) the current in the capacitor, (b) the current in the $5 \Omega$ resistance, (c) the active power output of the $15\angle0^\circ$ V source, and (d) the magnitude of the p.d. across the $j2 \Omega$ inductance.
\[(a) \, 1.03 \, \text{A} \quad (b) \, 1.48 \, \text{A} \\
(c) \, 16.28 \, \text{W} \quad (d) \, 3.47 \, \text{V} \]

8. A balanced 3-phase delta-connected load is shown in Figure 31.20. Use mesh-current analysis to determine the values of mesh currents
Figure 31.18

$I_1, I_2$ and $I_3$ shown and hence find the line currents $I_R, I_Y$ and $I_B$.

$[I_1 = 83\angle 173.13^\circ \text{ A}, I_2 = 83\angle 53.13^\circ \text{ A},
I_3 = 83\angle -66.87^\circ \text{ A}, I_R = 143.8\angle 143.13^\circ \text{ A},
I_Y = 143.8\angle 23.13^\circ \text{ A}, I_B = 143.8\angle -96.87^\circ \text{ A}]$

9 Use mesh-circuit analysis to determine the value of currents $I_A$ to $I_E$ in the circuit shown in Figure 31.21.

$[I_A = 2.40\angle 52.52^\circ \text{ A}; I_B = 1.02\angle 46.19^\circ \text{ A};
I_C = 1.39\angle 57.17^\circ \text{ A}; I_D = 0.67\angle 15.57^\circ \text{ A};
I_E = 0.996\angle 83.74^\circ \text{ A}]$

Figure 31.19

Figure 31.20

Figure 31.21

Figure 31.22

Nodal analysis

10 Repeat problems 1, 2, 5, 8 and 10 on page 542 of Chapter 30, and problems 2, 3, 5, and 9 above, using nodal analysis.
11 Determine for the network shown in Figure 31.22 the voltage at node 1 and the voltage $V_{AB}$
\[ V_1 = 59.0 \angle -28.92^\circ \text{ V}; \ V_{AB} = 45.3 \angle 10.89^\circ \text{ V} \]

12 Determine the voltage $V_{PQ}$ in the network shown in Figure 31.23.
\[ V_{PQ} = 55.87 \angle 50.60^\circ \text{ V} \]

13 Use nodal analysis to determine the currents $I_A$, $I_B$ and $I_C$ shown in the network of Figure 31.24.
\[ I_A = 1.21 \angle 150.96^\circ \text{ A}; \ I_B = 1.06 \angle -56.32^\circ \text{ A}; \]
\[ I_C = 0.55 \angle 32.01^\circ \text{ A} \]

14 For the network shown in Figure 31.25 determine (a) the voltages at nodes 1 and 2, (b) the current in the 40 $\Omega$ resistance, (c) the current in the 20 $\Omega$ resistance, and (d) the magnitude of the active power dissipated in the 10 $\Omega$ resistance
\[ (a) \ V_1 = 88.12 \angle 33.86^\circ \text{ V}, \ V_2 = 58.72 \angle 72.28^\circ \text{ V} \]
\[ (b) \ 2.20 \angle 33.86^\circ \text{ A}, \text{ away from node 1}, \]
\[ (c) \ 2.80 \angle 118.65^\circ \text{ A}, \text{ away from node 1}, \ (d) \ 223 \text{ W} \]

15 Determine the voltage $V_{AB}$ in the network of Figure 31.26, using nodal analysis.
\[ V_{AB} = 54.23 \angle -102.52^\circ \text{ V} \]