# 32 The superposition theorem

At the end of this chapter you should be able to:

- solve d.c. and a.c. networks using the superposition theorem

## 32.1 Introduction

The superposition theorem states:

*'In any network made up of linear impedances and containing more than one source of e.m.f. the resultant current flowing in any branch is the phasor sum of the currents that would flow in that branch if each source were considered separately, all other sources being replaced at that time by their respective internal impedances.'*

## 32.2 Using the superposition theorem

The superposition theorem, which was introduced in Chapter 13 for d.c. circuits, may be applied to both d.c. and a.c. networks. A d.c. network is shown in Figure 32.1 and will serve to demonstrate the principle of application of the superposition theorem.

To find the current flowing in each branch of the circuit, the following six-step procedure can be adopted:

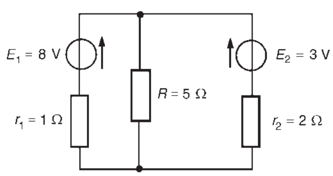


**Figure 32.1**

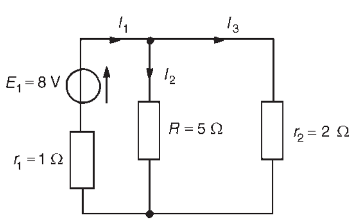


**Figure 32.2**

(i) Redraw the original network with one of the sources, say $E_2$, removed and replaced by $r_2$ only, as shown in Figure 32.2.

(ii) Label the current in each branch and its direction as shown in Figure 32.2, and then determine its value. The choice of current direction for $I_1$ depends on the source polarity which, by convention, is taken as flowing from the positive terminal as shown.

$R$ in parallel with $r_2$ gives an equivalent resistance of

$$(5 \times 2)/(5 + 2) = 10/7 = 1.429\ \Omega$$

as shown in the equivalent network of Figure 32.3. From Figure 28.3,

$$\text{current } I_1 = \frac{E_1}{(r_1 + 1.429)} = \frac{8}{2.429} = 3.294\ \text{A}$$

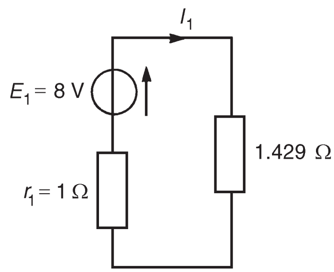


**Figure 32.3**

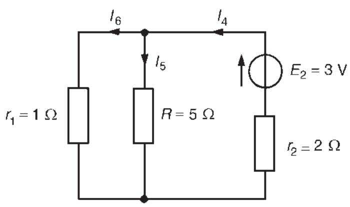


**Figure 32.4**

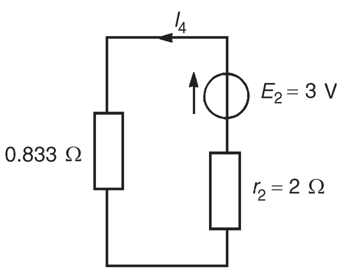


**Figure 32.5**

From Figure 32.2,

$$\text{current } I_2 = \left(\frac{r_2}{R + r_2}\right)(I_1) = \left(\frac{2}{5+2}\right)(3.294) = 0.941 \text{ A}$$

$$\text{and current } I_3 = \left(\frac{5}{5+2}\right)(3.294) = 2.353 \text{ A}$$

(iii) Redraw the original network with source $E_1$ removed and replaced by $r_1$ only, as shown in Figure 32.4.

(iv) Label the currents in each branch and their directions as shown in Figure 32.4, and determine their values.

$R$ and $r_1$ in parallel gives an equivalent resistance of

$$(5 \times 1)/(5 + 1) = 5/6\ \Omega \text{ or } 0.833\ \Omega,$$

as shown in the equivalent network of Figure 32.5. From Figure 32.5,

$$\text{current } I_4 = \frac{E_2}{r_2 + 0.833} = \frac{3}{2.833} = 1.059 \text{ A}$$

From Figure 32.4,

$$\text{current } I_5 = \left(\frac{1}{1+5}\right)(1.059) = 0.177 \text{ A}$$

$$\text{and current } I_6 = \left(\frac{5}{1+5}\right)(1.059) = 0.8825 \text{ A}$$

(v) Superimpose Figure 32.2 on Figure 32.4, as shown in Figure 32.6.

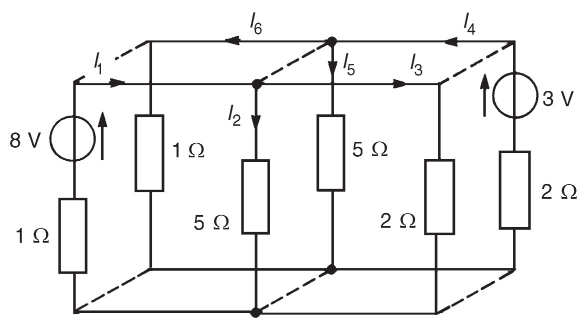


**Figure 32.6**

(vi) Determine the algebraic sum of the currents flowing in each branch. (Note that in an a.c. circuit it is the phasor sum of the currents that is required.)

From Figure 32.6, the resultant current flowing through the 8 V source is given by

$I_1 - I_6 = 3.294 - 0.882 = \mathbf{2.41\ A}$ (discharging, i.e., flowing from the positive terminal of the source).

The resultant current flowing in the 3 V source is given by

$I_3 - I_4 = 2.353 - 1.059 = \mathbf{1.29\ A}$ (charging, i.e., flowing into the positive terminal of the source).

The resultant current flowing in the 5 Ω resistance is given by

$I_2 + I_5 = 0.941 + 0.177 = \mathbf{1.12\ A}$

The values of current are the same as those obtained on page 536 by using Kirchhoff's laws.

The following problems demonstrate further the use of the superposition theorem in analysing a.c. as well as d.c. networks. The theorem is straightforward to apply, but is lengthy. Thévenin's and Norton's theorems (described in Chapter 33) produce results more quickly.

> Problem 1. A.c. sources of $100\angle 0^\circ$ V and internal resistance 25 Ω, and $50\angle 90^\circ$ V and internal resistance 10 Ω, are connected in parallel across a 20 Ω load. Determine using the superposition theorem, the current in the 20 Ω load and the current in each voltage source.

(This is the same problem as problem 1 on page 536 and problem 6 on page 553 and a comparison of methods may be made.)

The circuit diagram is shown in Figure 32.7. Following the above procedure:

(i) The network is redrawn with the $50\angle 90^\circ$ V source removed as shown in Figure 32.8

(ii) Currents $I_1$, $I_2$ and $I_3$ are labelled as shown in Figure 32.8.

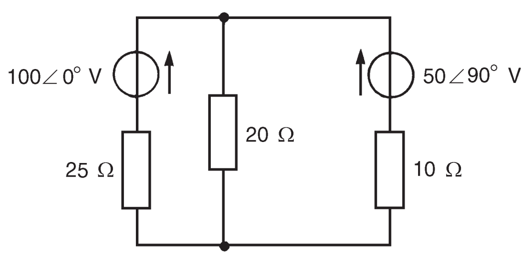


**Figure 32.7**

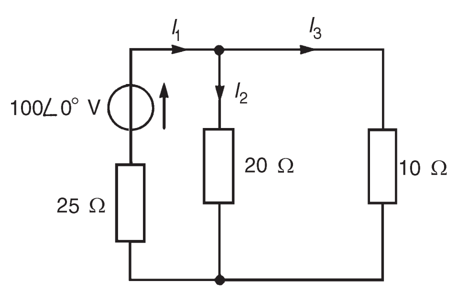


**Figure 32.8**

$$I_1 = \frac{100\angle 0^\circ}{25 + (10 \times 20)/(10 + 20)} = \frac{100\angle 0^\circ}{25 + 6.667} = 3.158\angle 0^\circ \text{ A}$$

$$I_2 = \left(\frac{10}{10 + 20}\right)(3.158\angle 0^\circ) = 1.053\angle 0^\circ \text{ A}$$

$$I_3 = \left(\frac{20}{10 + 20}\right)(3.158\angle 0^\circ) = 2.105\angle 0^\circ \text{ A}$$

(iii) The network is redrawn with the $100\angle 0^\circ$ V source removed as shown in Figure 32.9

(iv) Currents $I_4$, $I_5$ and $I_6$ are labelled as shown in Figure 32.9.

$$I_4 = \frac{50\angle 90^\circ}{10 + (25 \times 20)/(25 + 20)} = \frac{50\angle 90^\circ}{10 + 11.111}$$

$$= 2.368\angle 90^\circ \text{ A or } j2.368 \text{ A}$$

$$I_5 = \left(\frac{25}{20 + 25}\right)(j2.368) = j1.316 \text{ A}$$

$$I_6 = \left(\frac{20}{20 + 25}\right)(j2.368) = j1.052 \text{ A}$$

(v) Figure 32.10 shows Figure 32.9 superimposed on Figure 32.8, giving the currents shown.

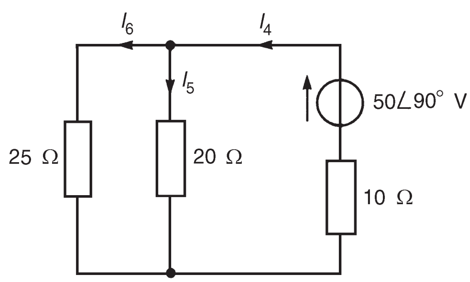


**Figure 32.9**

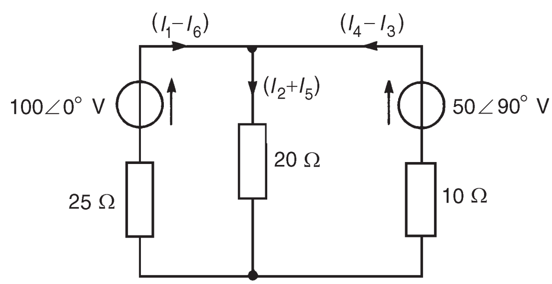


**Figure 32.10**

(vi) Current in the 20 Ω load, $I_2 + I_5 = (1.053 + j1.316)$ A or $\mathbf{1.69\angle 51.33^\circ}$ **A**

Current in the $100\angle 0^\circ$ V source, $I_1 - I_6 = (3.158 - j1.052)$ A or $\mathbf{3.33\angle -18.42^\circ}$ **A**

Current in the $50\angle 90^\circ$ V source, $I_4 - I_3 = (j2.368 - 2.105)$ or $\mathbf{3.17\angle 131.64^\circ}$ **A**

**Problem 2.** Use the superposition theorem to determine the current in the 4 Ω resistor of the network shown in Figure 32.11.

(i) Removing the 20 V source gives the network shown in Figure 32.12.

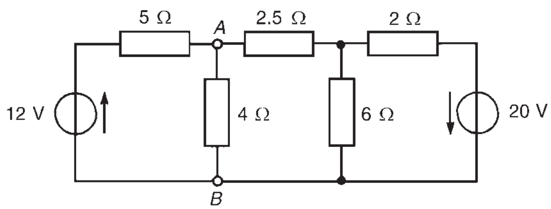


**Figure 32.11**

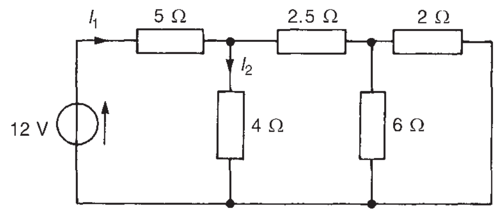


**Figure 32.12**

(ii) Currents $I_1$ and $I_2$ are shown labelled in Figure 32.12. It is unnecessary to determine the currents in all the branches since only the current in the 4 Ω resistance is required.

From Figure 32.12, 6 Ω in parallel with 2 Ω gives $(6 \times 2)/(6 + 2) = 1.5$ Ω, as shown in Figure 32.13. 2.5 Ω in series with 1.5 Ω gives 4 Ω, 4 Ω in parallel with 4 Ω gives 2 Ω, and 2 Ω in series with 5 Ω gives 7 Ω.

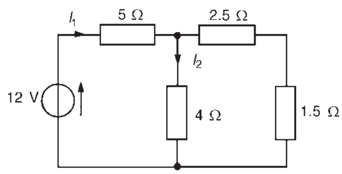


**Figure 32.13**

$$\text{Thus current } I_1 = \frac{12}{7} = 1.714 \text{ A and}$$

$$\text{current } I_2 = \left(\frac{4}{4+4}\right)(1.714) = 0.857 \text{ A}$$

(iii) Removing the 12 V source from the original network gives the network shown in Figure 32.14.

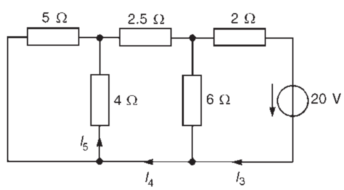


**Figure 32.14**

(iv) Currents $I_3$, $I_4$ and $I_5$ are shown labelled in Figure 32.14.

From Figure 32.14, 5 Ω in parallel with 4 Ω gives $(5 \times 4)/(5 + 4) = 20/9 = 2.222$ Ω, as shown in Figure 32.15, 2.222 Ω in series with 2.5 Ω gives 4.722 Ω, 4.722 Ω in parallel with 6 Ω gives $(4.722 \times 6)/(4.722 + 6) = 2.642$ Ω, 2.642 Ω in series with 2 Ω gives 4.642 Ω.

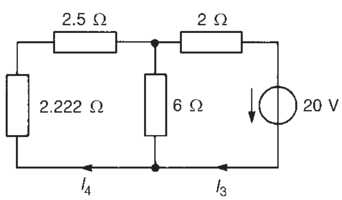


**Figure 32.15**

$$\text{Hence } I_3 = \frac{20}{4642} = 4.308 \text{ A}$$

$$I_4 = \left(\frac{6}{6+4.722}\right)(4.308) = 2.411 \text{ A, from Figure 32.15}$$

$$I_5 = \left(\frac{5}{4+5}\right)(2.411) = 1.339 \text{ A, from Figure 32.14}$$

(v) Superimposing Figure 32.14 on Figure 32.12 shows that the current flowing in the 4 Ω resistor is given by $I_5 - I_2$

(vi) $I_5 - I_2 = 1.339 - 0.857 =$ **0.48 A, flowing from B toward A** (see Figure 32.11)

Problem 3. Use the superposition theorem to obtain the current flowing in the $(4 + j3)\Omega$ impedance of Figure 32.16.

(i) The network is redrawn with $V_2$ removed, as shown in Figure 32.17.

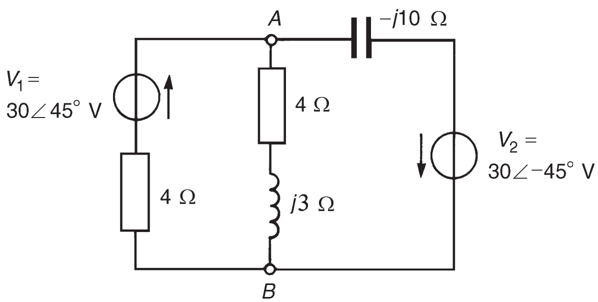


**Figure 32.16**

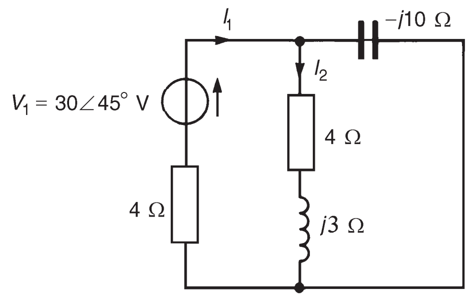


**Figure 32.17**

(ii) Current $I_1$ and $I_2$ are shown in Figure 32.17. From Figure 32.17, $(4 + j3)\Omega$ in parallel with $-j10\ \Omega$ gives an equivalent impedance of

$$\frac{(4 + j3)(-j10)}{(4 + j3 - j10)} = \frac{30 - j40}{4 - j7} = \frac{50\angle -53.13^\circ}{8.062\angle -60.26^\circ}$$

$$= 6.202\angle 7.13^\circ \text{ or } (6.154 + j0.770)\Omega$$

Total impedance of Figure 32.17 is

$$6.154 + j0.770 + 4 = (10.154 + j0.770)\Omega \text{ or } 10.183\angle 4.34^\circ\ \Omega$$

Hence current $I_1 = \dfrac{30\angle 45^\circ}{10.183\angle 4.34^\circ} = 2.946\angle 40.66^\circ$ A

and current $I_2 = \left(\dfrac{-j10}{4 - j7}\right)(2.946\angle 40.66^\circ)$

$$= \frac{(10\angle -90^\circ)(2.946\angle 40.66^\circ)}{8.062\angle -60.26^\circ}$$

$$= 3.654\angle 10.92^\circ \text{ A or } (3.588 + j0.692)\text{A}$$

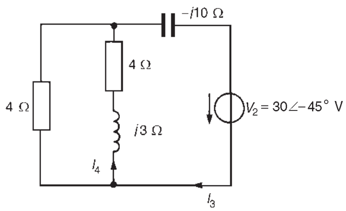


**Figure 32.18**

(iii) The original network is redrawn with $V_1$ removed, as shown in Figure 32.18.

(iv) Currents $I_3$ and $I_4$ are shown in Figure 32.18. From Figure 32.18, 4 Ω in parallel with $(4 + j3)\Omega$ gives an equivalent impedance of

$$\frac{4(4+j3)}{4+4+j3} = \frac{16+j12}{8+j3} = \frac{20\angle 36.87^\circ}{8.544\angle 20.56^\circ}$$
$$= 2.341\angle 16.31^\circ\ \Omega \text{ or } (2.247 + j0.657)\Omega$$

Total impedance of Figure 32.18 is

$$2.247 + j0.657 - j10 = (2.247 - j9.343)\Omega \text{ or } 9.609\angle -76.48^\circ\ \Omega$$

Hence current $I_3 = \dfrac{30\angle -45^\circ}{9.609\angle -76.48^\circ} = 3.122\angle 31.48^\circ$ A

and current $I_4 = \left(\dfrac{4}{8+j3}\right)(3.122\angle 31.48^\circ)$

$$= \frac{(4\angle 0^\circ)(3.122\angle 31.48^\circ)}{8.544\angle 20.56^\circ}$$
$$= 1.462\angle 10.92^\circ \text{ A or } (1.436 + j0.277)\text{A}$$

(v) If the network of Figure 32.18 is superimposed on the network of Figure 32.17, it can be seen that the current in the $(4 + j3)\Omega$ impedance is given by $I_2 - I_4$

(vi) $I_2 - I_4 = (3.588 + j0.692) - (1.436 + j0.277)$

$= \mathbf{(2.152 + j0.415)A}$ or $\mathbf{2.192\angle 10.92^\circ\ A}$, flowing from **A to B** in Figure 32.16.

Problem 4. For the a.c. network shown in Figure 32.19 determine, using the superposition theorem, (a) the current in each branch, (b) the magnitude of the voltage across the $(6 + j8)\Omega$ impedance, and (c) the total active power delivered to the network.

(a) (i) The original network is redrawn with $E_2$ removed, as shown in Figure 32.20.

(ii) Currents $I_1$, $I_2$ and $I_3$ are labelled as shown in Figure 32.20. From Figure 32.20, $(6 + j8)\Omega$ in parallel with $(2 - j5)\Omega$ gives an equivalent impedance of

$$\frac{(6+j8)(2-j5)}{(6+j8)+(2-j5)} = (5.123 - j3.671)\Omega$$

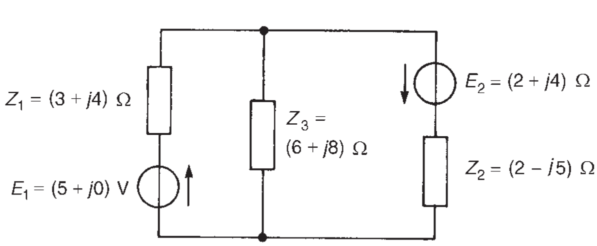


**Figure 32.19**

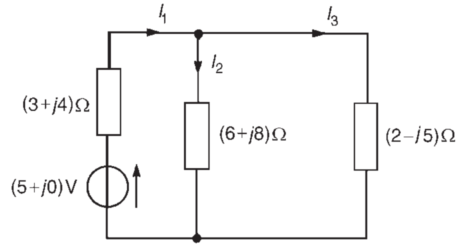


**Figure 32.20**

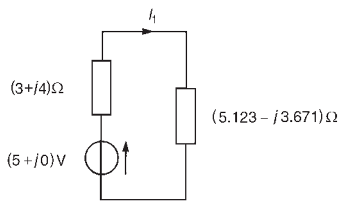


**Figure 32.21**

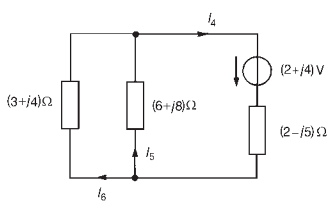


**Figure 32.22**

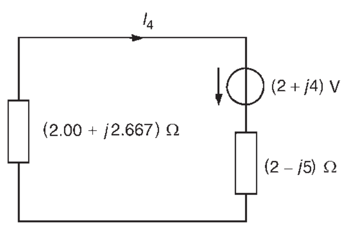


**Figure 32.23**

From the equivalent network of Figure 32.21,

$$\text{current } I_1 = \frac{5 + j0}{(3 + j4) + (5.123 - j3.671)}$$
$$= (0.614 - j0.025)\text{A}$$

$$\text{current } I_2 = \left[\frac{(2 - j5)}{(6 + j8) + (2 - j5)}\right](0.614 - j0.025)$$
$$= (-0.00731 - j0.388)\text{A}$$

$$\text{and current } I_3 = \left[\frac{(6 + j8)}{(6 + j8) + (2 - j5)}\right](0.614 - j0.025)$$
$$= (0.622 + j0.363)\text{A}$$

(iii) The original network is redrawn with $E_1$ removed, as shown in Figure 32.22.

(iv) Currents $I_4$, $I_5$ and $I_6$ are shown labelled in Figure 32.22 with $I_4$ flowing away from the positive terminal of the $(2 + j4)$V source.

From Figure 32.22, $(3 + j4)\Omega$ in parallel with $(6 + j8)\Omega$ gives an equivalent impedance of

$$\frac{(3 + j4)(6 + j8)}{(3 + j4) + (6 + j8)} = (2.00 + j2.667)\Omega$$

From the equivalent network of Figure 32.23,

$$\text{current } I_4 = \frac{(2 + j4)}{(2.00 + j2.667) + (2 - j5)}$$
$$= (-0.062 + j0.964)\text{A}$$

From Figure 32.22,

$$\text{current } I_5 = \left[\frac{(3 + j4)}{(3 + j4) + (6 + j8)}\right](-0.062 + j0.964)$$
$$= (-0.0207 + j0.321)\text{A}$$

$$P = (5)(0.843)\cos(47.16^\circ - 0^\circ)$$
$$+ (\sqrt{(2^2+4^2)})(1.440)\cos\left(67.12^\circ - \arctan\tfrac{4}{2}\right)$$
$$= 2.866 + 6.427 = 9.293 \text{ W}$$
$$= \mathbf{9.3\ W}, \text{ correct to one dec. place.}$$

(This value may be checked since total active power dissipated is given by:

$$P = (I_1 + I_6)^2(3) + (I_2 - I_5)^2(6) + (I_3 + I_4)^2(2)$$
$$= (0.843)^2(3) + (0.709)^2(6) + (1.440)^2(2)$$
$$= 2.132 + 3.016 + 4.147 = 9.295 \text{ W}$$
$$= \mathbf{9.3\ W,} \text{ correct to one dec. place.)}$$

Problem 5. Use the superposition theorem to determine, for the network shown in Figure 32.25, (a) the magnitude of the current flowing in the capacitor, (b) the p.d. across the 5 Ω resistance, (c) the active power dissipated in the 20 Ω resistance and (d) the total active power taken from the supply.

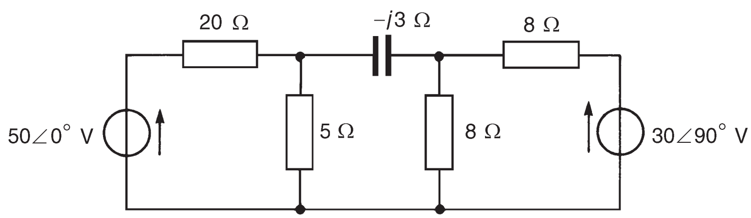


**Figure 32.25**

(i) The network is redrawn with the 30∠90° V source removed, as shown in Figure 32.26.

(ii) Currents $I_1$ to $I_5$ are shown labelled in Figure 32.26. From Figure 32.26, two 8 Ω resistors in parallel give an equivalent resistance of 4 Ω.

$$\text{Hence } I_1 = \frac{50\angle 0^\circ}{20 + (5(4 - j3)/(5 + 4 - j3))} = 2.220\angle 2.12^\circ \text{ A}$$

$$I_2 = \frac{(4 - j3)}{(5 + 4 - j3)} I_1 = 1.170\angle{-16.32^\circ} \text{ A}$$

$$I_3 = \left(\frac{5}{5 + 4 - j3}\right) I_1 = 1.170\angle 20.55^\circ \text{ A}$$

$$I_4 = \left(\frac{8}{8 + 8}\right) I_3 = 0.585\angle 20.55^\circ \text{ A} = I_5$$

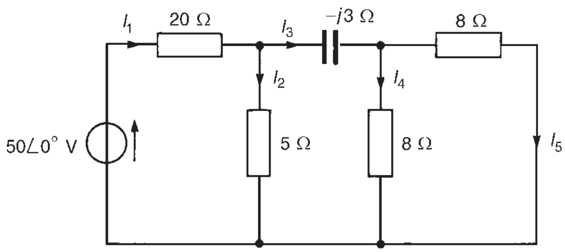


Figure 32.26

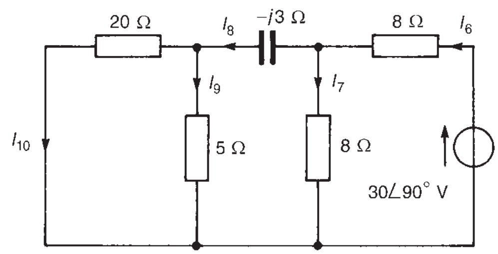


Figure 32.27

(iii) The original network is redrawn with the $50\angle 0^\circ$ V source removed, as shown in Figure 32.27.

(iv) Currents $I_6$ to $I_{10}$ are shown labelled in Figure 32.27. From Figure 32.27, 20 Ω in parallel with 5 Ω gives an equivalent resistance of $(20 \times 5)/(20+5) = 4\ \Omega$.

$$\text{Hence}\quad I_6 = \frac{30\angle 90^\circ}{8 + (8(4-j3)/(8+4-j3))} = 2.715\angle 96.52^\circ\ \text{A}$$

$$I_7 = \frac{(4-j3)}{(8+4-j3)} I_6 = 1.097\angle 73.69^\circ\ \text{A}$$

$$I_8 = \left(\frac{8}{8+4-j3}\right) I_6 = 1.756\angle 110.56^\circ\ \text{A}$$

$$I_9 = \left(\frac{20}{20+5}\right) I_8 = 1.405\angle 110.56^\circ\ \text{A}$$

$$\text{and}\quad I_{10} = \left(\frac{5}{20+5}\right) I_8 = 0.351\angle 110.56^\circ\ \text{A}$$

(a) The current flowing in the capacitor is given by

$$(I_3 - I_8) = 1.170\angle 20.55^\circ - 1.756\angle 110.56^\circ$$
$$= (1.712 - j1.233)\text{A or } 2.11\angle -35.76^\circ\ \text{A}$$

i.e., **the magnitude of the current in the capacitor is 2.11 A**

(b) The p.d. across the 5 Ω resistance is given by $(I_2 + I_9)$ (5).

$$(I_2 + I_9) = 1.170\angle -16.32^\circ + 1.405\angle 110.56^\circ$$
$$= (0.629 + j0.987)\text{A or } 1.17\angle 57.49^\circ\ \text{A}$$

Hence **the magnitude of the pd. across the 5 Ω resistance** is $(1.17)(5) =$ **5.85 V**

(c) Active power dissipated in the 20 Ω resistance is given by $(I_1 - I_{10})^2(20)$.

$$(I_1 - I_{10}) = 2.220\angle 2.12^\circ - 0.351\angle 110.56^\circ$$
$$= (2.342 - j0.247)\text{A or } 2.355\angle -6.02^\circ\ \text{A}$$

Hence **the active power dissipated in the 20 Ω resistance** is $(2.355)^2(20) = \mathbf{111\ W}$

(d) Active power developed by the $50\angle 0^\circ$ V source

$$P_1 = V(I_1 - I_{10})\cos\phi_1 = (50)(2.355)\cos(6.02^\circ - 0^\circ)$$
$$= 117.1\ \text{W}$$

Active power developed by $30\angle 90$ V source,

$$P_2 = 30(I_6 - I_5)\cos\phi_2$$
$$(I_6 - I_5) = 2.715\angle 96.52^\circ - 0.585\angle 20.55^\circ$$
$$= (-0.856 + j2.492)\text{A or } 2.635\angle 108.96^\circ\ \text{A}$$

Hence $P_2 = (30)(2.635)\cos(108.96^\circ - 90^\circ) = 74.8$ W.

**Total power developed,** $P = P_1 + P_2 = 117.1 + 74.8 = \mathbf{191.9\ W}$

(This value may be checked by summing the $I^2R$ powers dissipated in the four resistors.)

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*Further problems on the superposition theorem may be found in Section 32.3 following, problems 1 to 8.*

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## 32.3 Further problems on the superposition theorem

1 Repeat problems 1, 5, 8 and 9 on page 542, of Chapter 30, and problems 3, 5 and 13 on page 559, of Chapter 31, using the superposition theorem.

2 Two batteries each of e.m.f. 15 V are connected in parallel to supply a load of resistance 2.0 Ω. The internal resistances of the batteries are 0.5 Ω and 0.3 Ω. Determine, using the superposition theorem, the current in the load and the current supplied by each battery. [6.86 A; 2.57 A; 4.29 A]

3 Use the superposition theorem to determine the magnitude of the current flowing in the capacitive branch of the network shown in Figure 32.28. [2.584 A]

4 A.c. sources of $20\angle 90^\circ$ V and internal resistance 10 Ω and $30\angle 0^\circ$ V and internal resistance 12 Ω are connected in parallel across an 8 Ω load. Use the superposition theorem to determine (a) the current in the 8 Ω load, and (b) the current in each voltage source. [(a) 1.30 A (b) $20\angle 90^\circ$ V source discharges at $1.58\angle 120.98^\circ$ A, $30\angle 0^\circ$ V source discharges at $1.90\angle -16.49^\circ$ A]

5 Use the superposition theorem to determine current $I_x$ flowing in the 5 Ω resistance of the network shown in Figure 32.29. [$0.529\angle 5.71^\circ$ A]

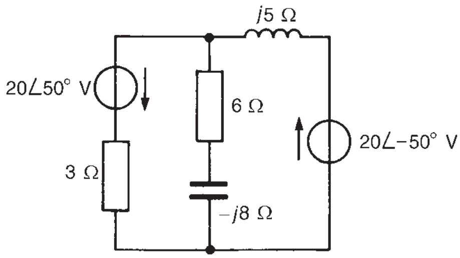


Figure 32.28

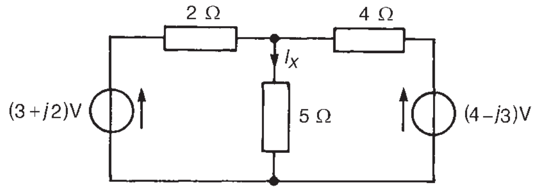


Figure 32.29

6 For the network shown in Figure 32.30, determine, using the superposition theorem, (a) the current flowing in the capacitor, (b) the current flowing in the 2 Ω resistance, (c) the p.d. across the 5 Ω resistance, and (d) the total active circuit power.
[(a) 1.28 A (b) 0.74 A (c) 3.01 V (d) 2.91 W]

7 (a) Use the superposition theorem to determine the current in the 12 Ω resistance of the network shown in Figure 32.31. Determine also the p.d. across the 8 Ω resistance and the power dissipated in the 20 Ω resistance.
(b) If the 37.5 V source in Figure 32.31 is reversed in direction, determine the current in the 12 Ω resistance.
[(a) 0.375 A, 8.0 V, 57.8 W (b) 0.625 A]

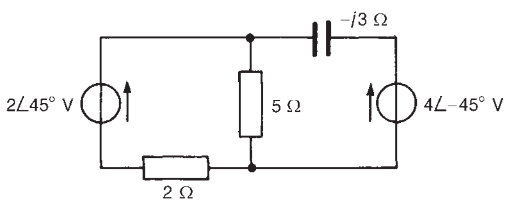


Figure 32.30

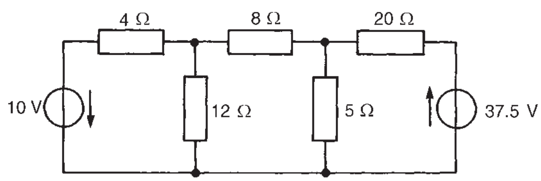


Figure 32.31

8 For the network shown in Figure 32.32, use the superposition theorem to determine (a) the current in the capacitor, (b) the pd. across the 10 Ω resistance, (c) the active power dissipated in the 20 Ω resistance, and (d) the total active circuit power.
[(a) 3.97 A (b) 28.7 V (c) 36.4 W (d) 371.6 W]

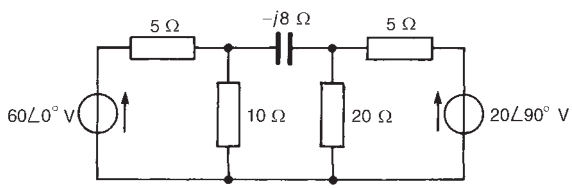


Figure 32.32