Quantization and Coding

Quantization and Preprocessing

After reading this lesson, you will learn about

> Need for preprocessing before quantization;

Introduction

In this module we shall discuss about a few aspects of analog to digital (A/D) conversion as relevant for the purpose of coding, multiplexing and transmission. The basic principles of analog to digital conversion will not be discussed. Subsequently we will discuss about several lossy coding and compression techniques such as the pulse code modulation (PCM), differential pulse code modulation (DPCM), and delta modulation (DM). The example of telephone grade speech signal having a narrow bandwidth of 3.1 kHz (from 300 Hz to 3.4 kHz) will be used extensively in this module.

Need for Preprocessing Before Digital Conversion

It is easy to appreciate that the electrical equivalent of human voice is summarily a random signal, **Fig.3.10.1**. It is also well known that the bandwidth of an audible signal (voice, music etc.) is less than 20 KHz (typical frequency range is between 20 Hz and 20KHz). Interestingly, the typical bandwidth of about 20 KHz is not considered for designing a telephone communication system. Most of the voice signal energy is limited within 3.4 KHz. While a small amount of energy beyond 3.4 KHz adds to the quality of voice, the two important features of a) message intelligibility and b) speaker recognition are retained when a voice signal is band limited to 3.4 KHz. This band limited voice signal is commonly referred as 'telephone grade speech signal'.



Fig. 3.10.1 Sketch of random speech signal vs. time

A very popular ITU-T (International Telecommunication Union) standard specifies the bandwidth of telephone grade speech signal between 300 Hz and 3.4 kHz. The lower cut off frequency of 300 Hz has been chosen instead of 20 Hz for multiple practical reasons. The power line frequency (50Hz) is avoided. Further the physical size and cost of signal processing elements such as transformer and capacitors are also suitable for the chosen lower cut-off frequency of 300 Hz. A very important purpose of squeezing the bandwidth is to allow a large number of speakers to communicate simultaneously through a telephone network while sharing costly trunk lines using the principle of multiplexing. A standard rate of sampling for telephone grade speech signal of one speaker is 8-Kilo samples/ sec (Ksps).

Usually, an A/D converter quantizes an input signal properly if the signal is within a specified range. As speech is also a random signal, there is always a possibility that the amplitude of speech signal at the input of a quantizer goes beyond this range. If no protection is taken for this problem and if the probability of such event is not negligible, even a good quantizer will lead to unacceptably distorted version of the signal. A possible remedy of this problem is to (a) study and assess the variance of random speech signal amplitude and (b) to adjust the signal variance within a reasonable limit. This is often ensured by using a variance estimation unit and a variable gain amplifier, Fig. 3.10.2. Another preprocessing which may be necessary is of DC adjustment of the input speech signal. To explain this point, let us assume that the input range of a quantizer is ' \pm V' volts. This quantizer expects an input signal whose DC (i.e average) value is zero. However if the input signal has an unwanted dc bias (\overline{x} in Fig 3.10.2) this also should be removed. For precise quantization, a mean estimation unit can be used to estimate a local mean and subtract it from the input signal. If both the processes of mean removal and variance normalization are to be adopted, the mean of the signal should be adjusted first.



Fig. 3.10.2 Scheme for mean removal and variance normalization

Pulse Code Modulation

After reading this lesson, you will learn about

- Principle of Pulse Code Modulation;
- > Signal to Quantization Noise Ratio for uniform quantizaton;

A schematic diagram for Pulse Code Modulation is shown in **Fig 3.11.1**. The analog voice input is assumed to have zero mean and suitable variance such that the signal samples at the input of A/D converter lie satisfactorily within the permitted single range. As discussed earlier, the signal is band limited to 3.4 KHz by the low pass filter.



Fig. 3.11.1 Schematic diagram of a PCM coder – decoder

Let x (t) denote the filtered telephone-grade speech signal to be coded. The process of analog to digital conversion primarily involves three operations: (a) Sampling of x (t), (b) Quantization (i.e. approximation) of the discrete time samples, x (kT_s) and (c) Suitable encoding of the quantized time samples x_q (kT_s). T_s indicates the sampling interval where $R_s = 1/T_s$ is the sampling rate (samples /sec). A standard sampling rate for speech signal, band limited to 3.4 kHz, is 8 Kilo-samples per second (T_s = 125 μ sec), thus, obeying Nyquist's sampling theorem. We assume instantaneous sampling for our discussion. The encoder in **Fig 3.11.1** generates a group of bits representing one quantized sample. A parallel-to-serial (P/S) converter is optionally used if a serial bit stream is desired at the output of the PCM coder. The PCM coded bit stream may be taken for further digital signal processing and modulation for the purpose of transmission.

The PCM decoder at the receiver expects a serial or parallel bit-stream at its input so that it can decode the respective groups of bits (as per the encoding operation) to generate quantized sample sequence $[x'_q (kT_s)]$. Following Nyquist's sampling theorem for band limited signals, the low pass filter produces a close replica $\hat{x}(t)$ of the original speech signal x (t).

If we consider the process of sampling to be ideal (i.e. instantaneous sampling) and if we assume that the same bit-stream as generated by PCM encoder is available at PCM decoder, we should still expect $\hat{x}(t)$ to be somewhat different from x (t). This is

solely because of the process of quantization. As indicated, quantization is an approximation process and thus, causes some distortion in the reconstructed analog signal. We say that quantization contributes to "noise". The issue of quantization noise, its characterization and techniques for restricting it within an acceptable level are of importance in the design of high quality signal coding and transmission system. We focus a bit more on a performance metric called SQNR (Signal to Quantization Noise power Ratio) for a PCM codec. For simplicity, we consider uniform quantization process. The input-output characteristic for a uniform quantizer is shown in **Fig 3.11.2(a)**. The input signal range (\pm V) of the quantizer has been divided in eight equal intervals. The width of each interval, δ , is known as the step size. While the amplitude of a time sample x (kT_s) may be any real number between +V and -V, the quantizer presents only one of the allowed eight values ($\pm \delta$, $\pm 3\delta/2$, ...) depending on the proximity of x (kT_s) to these levels.



Fig 3.11.2(a) Linear or uniform quantizer

The quantizer of **Fig 3.11.2(a)** is known as "mid-riser" type. For such a mid-riser quantizer, a slightly positive and a slightly negative values of the input signal will have different levels at output. This may be a problem when the speech signal is not present but small noise is present at the input of the quantizer. To avoid such a random fluctuation at the output of the quantizer, the "mid-tread" type uniform quantizer [**Fig 3.11.2(b**)] may be used.



Fig 3.11.2(b) Mid-tread type uniform quantizer characteristics

SQNR for uniform quantizer

In **Fig.3.11.1** $x(kT_s)$ represents a discrete time $(t = kT_s)$ continuous amplitude sample of x(t) and $x_q(kT_s)$ represents the corresponding quantized discrete amplitude value. Let e_k represents the error in quantization of the k^{th} sample i.e.

$$e_k = x_q(kT_s) - x(kT_s)$$
 3.11.1

Let,

M = Number of permissible levels at the quantizer output.

N = Number of bits used to represent each sample.

 $\pm V$ = Permissible range of the input signal x (t).

Hence,

 $M=2^{N}$ and, $M.\delta \cong 2.V$ [Considering large M and a mid-riser type quantizer]

Let us consider a small amplitude interval dx such that the probability density function (pdf) of x(t) within this interval is p(x). So, p(x)dx is the probability that x(t) lies in the range $(x - \frac{dx}{2})$ and $(x + \frac{dx}{2})$. Now, an expression for the mean square quantization error $\overline{e^2}$ can be written as:

$$\overline{e^2} = \int_{x_1 - \delta/2}^{x_1 + \delta/2} p(x)(x - x_1)^2 dx + \int_{x_2 - \delta/2}^{x_2 + \delta/2} p(x)(x - x_2)^2 dx + \dots$$
3.11.2

For large M and small δ we may fairly assume that p(x) is constant within an interval, i.e. $p(x) = p_1$ in the 1st interval, $p(x) = p_2$ in the 2nd interval, ..., $p(x) = p_k$ in the kth interval.

Therefore, the previous equation can be written as

$$\overline{e^2} = (p_1 + p_2 + \dots) \int_{-\delta/2}^{\delta/2} y^2 dy$$

Where, $y = x - x_k$ for all 'k'

So,

$$\overline{e^2} = (p_1 + p_2 + \dots) \frac{\delta^3}{12}$$
$$= [(p_1 + p_2 + \dots)\delta] \frac{\delta^2}{12}$$

Now, note that $(p_1 + p_2 + \dots + p_k + \dots)\delta = 1.0$ $\therefore \overline{e^2} = \frac{\delta^2}{12}$

The above mean square error represents power associated with the random error signal. For convenience, we will also indicate it as N_Q .

Calculation of Signal Power (S_i)

After getting an estimate of quantization noise power as above, we now have to find the signal power. In general, the signal power can be assessed if the signal statistics (such as the amplitude distribution probability) is known. The power associated with x (t) can be expressed as

$$S_{i} = \overline{\chi^{2}(t)} = \int_{-V}^{+V} \chi^{2}(t) p(x) dx$$

where p (x) is the pdf of x (t). In absence of any specific amplitude distribution it is common to assume that the amplitude of signal x (t) is uniformly distributed between $\pm V$.

In this case, it is easy to see that

$$S_{i} = \overline{\chi^{2}(t)} = \int_{-V}^{+V} \chi^{2}(t) \frac{1}{2V} dx = \left[\frac{\chi^{3}}{3.2V}\right]_{-V}^{+V} = \frac{V^{2}}{3} = \frac{\left(M\delta\right)^{2}}{12}$$

Now the SNR can be expressed as,

$$\frac{S_i}{N_Q} = \frac{\frac{V^2}{3}}{\frac{\delta^2}{12}} = \frac{\left(M\delta\right)^2}{\frac{12}{12}} = M^2$$

It may be noted from the above expression that this ratio can be increased by increasing the number of quantizer levels N.

Also note that S_i is the power of x (t) at input of the sampler and hence, may not represent the SQNR at the output of the low pass filter in PCM decoder. However, for large N, small δ and ideal and smooth filtering (e.g. Nyquist filtering) at the PCM

decoder, the power S_o of desired signal at the output of the PCM decoder can be assumed to be almost the same as S_i i.e.,

$$S_o \simeq S_i$$

With this justification the SQNR at the output of a PCM codec, can be expressed as,

$$SQNR = \frac{S_o}{N_Q} \approx M^2 = (2^N)^2 = 4^N$$
$$\frac{S_o}{N_Q}\Big|_{dB>} = 10\log_{10}\left(\frac{S_o}{N_Q}\right) \approx 6.02NdB$$

and in dB,

A few observations

- (a) Note that if actual signal excursion range is less than $\pm V$, S_o / N_o < 6.02NdB.
- (b) If one quantized sample is represented by 8 bits after encoding i.e., N = 8, $SQNR \simeq 48dB$.
- (c) If the amplitude distribution of x (t) is not uniform, then the above expression may not be applicable.

Problems

- Q3.11.1) If a sinusoid of peak amplitude 1.0V and of frequency 500Hz is sampled at 2 k-sample /sec and quantized by a linear quantizer, determine SQNR in dB when each sample is represented by 6 bit.
- Q3.11.2) How much is the improvement in SQNR of problem 3.11.1 if each sample is represented by 10 bits?
- Q3.11.3) What happens to SQNR of problem 3.11.2 if each sampling rate is changed to 1.5 k-samples/ sec?

Logarithmic Pulse Code Modulation (Log PCM) and Companding

After reading this lesson, you will learn about:

- Reason for logarithmic PCM;
- > A-law and μ -law Companding;

In a linear or uniform quantizer, as discussed earlier, the quantization error in the k-th sample is

$$e_k = x (t) - x_q (kT_s)$$
 3.12.1

and the maximum error magnitude in a quantized sample is,

$$Max \left| \boldsymbol{e}_{k} \right| = \frac{\delta}{2}$$
 3.12.2

So, if x (t) itself is small in amplitude and such small amplitudes are more probable in the input signal than amplitudes closer to ' \pm V', it may be guessed that the quantization noise of such an input signal will be significant compared to the power of x (t). This implies that SQNR of usually low signal will be poor and unacceptable. In a practical PCM codec, it is often desired to design the quantizer such that the SQNR is almost independent of the amplitude distribution of the analog input signal x (t).

This is achieved by using a non-uniform quantizer. A non-uniform quantizer ensures smaller quantization error for small amplitude of the input signal and relatively larger step size when the input signal amplitude is large. The transfer characteristic of a non – uniform quantizer has been shown in **Fig 3.12.1**. A non-uniform quantizer can be considered to be equivalent to an amplitude pre-distortion process [denoted by y = c(x) in **Fig 3.12.2**] followed by a uniform quantizer with a fixed step size ' δ '. We now briefly discuss about the characteristics of this pre-distortion or 'compression' function y = c(x).



Fig 3.12.1 Transfer characteristic of a non-uniform quantizer



Fig. 3.12.2 An equivalent form of a non-uniform quantizer

Mathematically, c (x) should be a monotonically increasing function of 'x' with odd symmetry **Fig 3.12.3**. The monotonic property ensures that $c^{-1}(x)$ exists over the range of 'x(t)' and is unique with respect to c (x) i.e., $c(x) \times c^{-1}(x) = 1$.



Fig. 3.12.3 A desired transfer characteristic for non-linear quantization process

Remember that the operation of $c^{-1}(x)$ is necessary in the PCM decoder to get back the original signal undistorted. The property of odd symmetry i.e., c(-x) = -c(x) simply takes care of the full range ' \pm V' of x (t). The range ' \pm V' of x (t) further implies the following:

$$c (x) = +V, for x = +V;= 0, for x = 0;= -V, for x = -V; 3.12.3$$

Let the k-th step size of the equivalent non-linear quantizer be ' δ_k ' and the number of signal intervals be 'M'. Further let the k-th representation level after quantization when the input signal lies between ' x_k ' and ' x_{k+1} ' be ' y_k ' where

$$y_k = \frac{1}{2} (x_k + x_{k+1}), k = 0, 1, \dots, (M-1)$$
 3.12.4

The corresponding quantization error 'ek' is

$$e_k = x - y_k \; ; \quad x_k < x \leq x_{k+1}$$

Now observe from **Fig 3.12.3** that ' δ_k ' should be small if ' $\frac{dc(x)}{dx}$ ' i.e., the slope of y = c(x) is large.

In view of this, let us make the following simple approximation on c (x):

$$\frac{dc(x)}{dx} \approx \frac{2V}{M} \frac{1}{\delta_k}, \qquad k = 0, 1, \dots, (M-1) \qquad 3.12.5$$
$$\delta_k = \chi_{k+1} - \chi_k, \qquad k = 0, 1, \dots, (M-1)$$

and

Note that, $\frac{2V}{M}$ is the fixed step size of the uniform quantizer Fig. 3.12.2.

Let us now assume that the input signal is zero mean and its pdf p(x) is symmetric about zero. Further for large number of intervals we may assume that in each interval I_k, k = 0,1,...,(M-1), the p(x) is constant. So if the input signal x (t) is between x_k and x_{k+1} , i.e., $x_k < x \le x_{k+1}$,

$$p(x) \simeq p(y_k)$$

So, the probability that x lies in the k-th interval I_k ,

$$I_{k} = p_{k} \triangleq P_{r}(x_{k} < x \le x_{k+1}) = p(y_{k})\delta_{k}$$
where,
$$\sum_{0}^{M-1} P_{r}(x_{k} < x \le x_{k+1}) = 1$$

Now, the mean square quantization error e^2 can be determined as follows:

$$\overline{e}^{2} = \int_{-V}^{+V} (x - y_{k})^{2} p(x) dx$$

$$= \sum_{k=0}^{M-1} \int_{x_{k}}^{x_{k+1}} (x - y_{k})^{2} p(y_{k}) dx$$

$$= \sum_{k=0}^{M-1} \frac{p_{k}}{\delta_{k}} \int_{x_{k}}^{x_{k+1}} (x - y_{k})^{2} dx$$

$$= \sum_{k=0}^{M-1} \frac{p_{k}}{\delta_{k}} \frac{1}{3} \left[(x_{k+1} - y_{k})^{3} - (x_{k} - y_{k})^{3} \right]$$

$$= \sum_{k=0}^{M-1} \frac{1}{3} \left(\frac{p_{k}}{\delta_{k}} \right) \left\{ \left[x_{k+1} - \frac{1}{2} (x_{k} + x_{k+1}) \right]^{3} - \left[x_{k} - \frac{1}{2} (x_{k} + x_{k+1}) \right]^{3} \right\}$$

$$=\frac{1}{3}\sum_{k=0}^{M-1}\frac{p_{k}}{\delta_{k}}\frac{1}{4}\delta_{k}^{3}=\frac{1}{12}\sum_{k=0}^{M-1}p_{k}\delta_{k}^{2}$$
3.12.7

Now substituting

$$\delta_k \simeq \frac{2V}{M} \left[\frac{dc(x)}{dx} \right]^{-1}$$

in the above expression, we get an approximate expression for mean square error as

$$\overline{e^{2}} = \frac{V^{2}}{3M^{2}} \sum_{k=0}^{M-1} p_{k} \left[\frac{dc(x)}{dx} \right]^{-2}$$
 3.12.8

The above expression implies that the mean square error due to non-uniform quantization can be expressed in terms of the continuous variable x, -V < x < +V, and having a pdf p (x) as below:

$$\overline{e^2} \approx \frac{V^2}{{}^3M^2} \int_{-V}^{+V} p(x) \left[\frac{dc(x)}{dx} \right]^{-2} dx \qquad 3.12.9$$

Now, we can have an expression of SQNR for a non-uniform quantizer as: V

$$SQNR \simeq \left(\frac{3M^2}{V^2}\right) \frac{\int_{-V}^{+V} x^2 p(x) dx}{\int_{-V}^{+V} p(x) \left[\frac{dc(x)}{dx}\right]^2}$$
3.12.10

The above expression is important as it gives a clue to the desired form of the compression function y = c(x) such that the SQNR can be made largely independent of the pdf of x (t).

It is easy to see that a desired condition is:

$$\frac{dc(x)}{dx} = \frac{K}{x} \quad \text{where } -V < x < +V \text{ and } K \text{ is a positive constant.}$$

i.e.,
$$c(x) = V + K \ln\left(\frac{x}{V}\right)$$
 for x > 0 3.12.11

and
$$c(x) = -c(x)$$
 3.12.12

Note:

Let us observe that $c(x) \rightarrow \pm \infty$ as $x \rightarrow 0$ from other side. Hence the above c(x) is not realizable in practice. Further, as stated earlier, the compression function c(x) must pass through the origin, i.e., c(x) = 0, for x = 0. This requirement is forced in a compression function in practical systems.

There are two popular standards for non-linear quantization known as

- (a) The μ law companding
- (b) The A law companding.

The μ - law has been popular in the US, Japan, Canada and a few other countries while the A - law is largely followed in Europe and most other countries, including India, adopting ITU-T standards.

The compression function c (x) for μ - law companding is (Fig. 3.12.4 and Fig. 3.12.5):

$$\frac{c(|x|)}{V} = \frac{\ln\left(1 + \frac{\mu|x|}{V}\right)}{\ln(1+\mu)}, \qquad 0 \le \frac{|x|}{V} \le 1.0 \qquad 3.12.13$$

' μ ' is a constant here. The typical value of μ lies between 0 and 255. $\mu = 0$ corresponds to linear quantization.



Fig. 3.12.4 μ -law companding characteristics(mu = 100)



Fig. 3.12.5 μ -law companding characteristics (mu = 0, 100, 255)

The compression function c (x) for A - law companding is (**Fig. 3.12.6**): |x|

$$\frac{c(|x|)}{V} = \frac{A\frac{|x|}{V}}{1+\ln A} , \qquad 0 \le \frac{|x|}{V} \le \frac{1}{A}$$
$$= \frac{1+\ln\left(A\frac{|x|}{V}\right)}{1+\ln A} , \qquad \frac{1}{A} \le \frac{|x|}{V} \le 1.0 \qquad 3.12.14$$

'A' is a constant here and the typical value used in practical systems is 87.5.



Fig. 3.12.6 *A-law companding characteristics* (*A* = 0, 87.5, 100, 255)

For telephone grade speech signal with 8-bits per sample and 8-Kilo samples per second, a typical SQNR of 38.4 dB is achieved in practice.

As approximately logarithmic compression function is used for linear quantization, a PCM scheme with non-uniform quantization scheme is also referred as "Log PCM" or "Logarithmic PCM" scheme.

Problems

- Q3.12.1) Consider Eq. 3.12.13 and sketch the compression of c (x) for μ = 50 and V = 2.0V
- Q3.12.2) Sketch the compression function c (x) for A law companding (Eq.3.12.14) when V = 1V and A = 50.
- Q3.12.3) Comment on the effectiveness of a non-linear quantizer when the peak amplitude of a signal is known to be considerably smaller than the maximum permissible voltage V.

Differential Pulse Code Modulation (DPCM)

After reading this lesson, you will learn about

- > Principles of DPCM;
- > DPCM modulation and de-modulation;
- Calculation of SQNR;
- One tap predictor;

The standard sampling rate for pulse code modulation (PCM) of telephone grade speech signal is $f_s = 8$ Kilo samples per sec with a sampling interval of 125 μ sec. Samples of this band limited speech signal are usually correlated as amplitude of speech signal does not change much within 125 μ sec. A typical auto correlation function R (τ) for speech samples at the rate 8 Kilo samples per sec is shown in **Fig 3.13.1**. R ($\tau = 125 \mu$ sec) is usually between 0.79 and 0.87. This aspect of speech signal is exploited in differential pulse code modulation (DPCM) technique. A schematic diagram for the basic DPCM modulator is shown in **Fig 3.13.2**. Note that a predictor block, a summing unit and a subtraction unit have been strategically added to the chain of blocks of PCM coder instead of feeding the sampler output x (kT_s) directly to a linear quantizer. An error sample e_p (kT_s) is fed.

The error sample is given by the following expression:

$$e_{p}(kT_{s}) = x(kT_{s}) - \hat{x}(kT_{s})$$
 3.13.1

 $\hat{x}(kT_s)$ is a predicted value for $x(kT_s)$ and is supposed to be close to $x(kT_s)$ such that $e_p(kT_s)$ is very small in magnitude. $e_p(kT_s)$ is called as the 'prediction error for the n-th sample'.



Fig. 3.13.1 Typical normalized auto-correlation coefficient for speech signal



Fig. 3.13.2 Schematic diagram of a DPCM modulator

If we assume a final enclosed bit rate of 64kbps as of a PCM coder, we envisage smaller step size for the linear quantizer compared to the step size of an equivalent PCM quantizer. As a result, it should be possible to achieve higher SQNR for DPCM codec delivering bits at the same rate as that of a PCM codec.

There is another possibility of decreasing the coded bit rate compared to a PCM system if an SQNR as achievable by a PCM codec with linear equalizer is sufficient. If the predictor output $\hat{x}(kT_s)$ can be ensured sufficiently close to $x(kT_s)$ then we can simply encode the quantizer output sample v(kT_s) in less than 8 bits. For example, if we choose to encode each of v(kT_s) by 6 bits, we achieve a serial bit rate of 48 kbps, which is considerably less than 64 Kbps. This is an important feature of DPCM, especially when the coded speech signal will be transmitted through wireless propagation channels.

We will now develop a simple analytical structure for a DPCM encoding scheme to bring out the role that nay be played by the prediction unit. As noted earlier,

$$e_p(kT_s) = k$$
-th input to quantizer $= x(kT_s) - \hat{x}(kT_s)$
 $\hat{x}(kT_s) = prediction of the k-th input sample $x(kT_s)$.
 $e_q(kT_s) = quantizer output for k-th prediction error.$
 $= c[e_p(kT_s)]$, where c[] indicates the transfer characteristic of the quantizer$

If $q(kT_s)$ indicates the quantization error for the k-th sample, it is easy to see that

$$e_q(kT_s) = e_p(kT_s) + q(kT_s)$$
 3.13.2

Further the input $u(kT_s)$ to the predictor is,

$$u(kT_{s}) = \hat{x}(kT_{s}) + e_{q}(kT_{s}) = \hat{x}(kT_{s}) + e_{p}(kT_{s}) + q(kT_{s})$$

= $x(kT_{s}) + q(kT_{s})$ 3.13.3

This equation shows that $u(kT_s)$ is indeed a quantized version of $x(kT_s)$. For a good prediction $e_p(kT_s)$ will usually be small compared to $x(kT_s)$ and $q(kT_s)$ in turn will be very small compared to. Hence, the predictor unit should be so designed that variance of $q(kT_s) < variance$ of $e_p(kT_s) << variance$ of $x(kT_s)$.

A block schematic diagram of a DPCM demodulator is shown in **Fig 3.13.3**. The scheme is straightforward and it tries to estimate $u(kT_s)$ using a predictor unit identical to the one used in the modulator. We have already observed that $u(kT_s)$ is very close to $x(kT_s)$ within a small quantization error of $q(kT_s)$. The analog speech signal is obtained by passing the samples $\hat{u}(kT_s)$ through an appropriate low pass filter. This low pass filter should have a 3 dB cut off frequency at 3.4kHz.



Fig. 3.13.3 Schematic diagram of a DPCM demodulator; note that the demodulator is very similar to a portion of the modulator

Calculation of SQNR for DPCM

The expression for signal to quantization noise power ratio for DPCM coding is:

$$SQNR = \frac{Variance \ of \ x(kT_s)}{Variance \ of \ q(kT_s)}$$
$$= \left[\frac{Variance \ of \ x(kT_s)}{Variance \ of \ e_p(kT_s)}\right] \cdot \left[\frac{Variance \ of \ e_p(kT_s)}{Variance \ of \ q(kT_s)}\right]$$

As in PCM coding, we are assuming instantaneous sampling and ideal low pass filtering. The first term in the above expression is the 'predictor gain (G_p) '. This gain visibly increases for better prediction, i.e., smaller variance of $e_p(kT_s)$. The second term, SNR_p is a property of the quantizer. Usually, a linear or uniform quantizer is used for simplicity in a DPCM codec. Good and efficient design of the predictor plays a crucial role in enhancing quality of signal or effectively reducing the necessary bit rate for transmission. In the following we shall briefly take up the example of a single-tap predictor.

Single-Tap Prediction

A single-tap predictor predicts the next input example $x(kT_s)$ from the immediate previous input sample $x([k-1]T_s)$.

Let, $\hat{x}(kT_s) = \hat{x}(k|k-1)$ = the k-th predicted sample, given the (k-1)th input sample = a.u(k-1|k-1)

Here, 'a' is known as the prediction co-efficient and u(k-1|k-1) is the (k-1)-th input to the predictor given the (k-1)-th input speech sample, i.e., x(k-1).

Now the k-th prediction error sample at the input of the quantizer may be expressed as $e_p(kT_s) \equiv e_p(k)$ $= x(k) - \hat{x}(k|k-1)$ = x(k) - au(k-1|k-1)3.13.4

The mean square error or variance of this prediction error samples is the statistical expectation of $e_p^2(k)$.

$$E[e_{p}^{2}(k)] = E[\{x(k) - a.u(k-1|k-1)\}^{2}]$$

= $E[x(k).x(k) - 2.a.x(k).u(k-1|k-1) + a^{2}.u(k-1|k-1).u(k-1|k-1)]$
= $E[x(k).x(k) - 2aE[x(k).u(k-1|k-1)] + a^{2}.E[u(k-1|k-1).u(k-1|k-1)]$
3.13.5

Let us note that E[x(k).x(k)]=R(0). Where $(R(\tau)$ indicates the autocorrelation coefficient. For the second term, let us assume that u(k-1|k-1) is an unbiased estimate of x(k-1) and that u(k-1|k-1) is a satisfactorily close estimate of x(k-1), so that we can use the following approximation:

$$E[x(k).u(k-1|k-1)] \simeq E[x(k).x(k-1)] = R(\tau = 1.T_s) \equiv R(1), say$$

The third term in the expanded form of
$$E[e_p^2(k)]$$
 can easily be identified as:
 $a^2 \cdot E[u(k-1|k-1) \cdot u(k-1|k-1)] = a^2 \cdot R(\tau = 0) = a^2 \cdot R(0)$
 $\therefore E[e_p^2(k)] = R(0) - 2 \cdot a \cdot R(1) + a^2 R(0)$
 $= R(0)[1 - 2a \cdot \frac{R(1)}{R(0)} + a^2]$
3.13.6

The above expression shows that the mean square error or variance of the prediction error can be minimized if a = R(1)/R(0).

Problems

- Q3.13.1) Is there any justification for DPCM, if the samples of a signal are known to be uncorrelated with each other?
- Q3.13.2) Determine the value of prediction co-efficient for one tap prediction unit if R(0) = 1.0 and R(1) = 0.9.

Delta Modulation (DM)

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After reading this lesson, you will learn about

- Principles and features of Delta Modulation;
- > Advantages and limitations of Delta Modulation;
- Slope overload distortion;
- ➢ Granular Noise;
- Condition for avoiding slope overloading;

If the sampling interval 'Ts' in DPCM is reduced considerably, i.e. if we sample a band limited signal at a rate much faster than the Nyquist sampling rate, the adjacent samples should have higher correlation (**Fig. 3.14.1**). The sample-to-sample amplitude difference will usually be very small. So, one may even think of only 1-bit quantization of the difference signal. The principle of Delta Modulation (DM) is based on this premise. Delta modulation is also viewed as a 1-bit DPCM scheme. The 1-bit quantizer is equivalent to a two-level comparator (also called as a hard limiter). **Fig. 3.14.2** shows the schematic arrangement for generating a delta-modulated signal. Note that,

$$e(kT_s) = x(kT_s) - \hat{x}(kT_s)$$
 3.14.1
= $x(kT_s) - u([k-1]T_s)$ 3.14.2



Fig. 3.14.1 The correlation increases when the sampling interval is reduced



Fig. 3.14.2 Block diagram of a delta modulator

Some interesting features of Delta Modulation

- No effective prediction unit the prediction unit of a DPCM coder (**Fig. 3.13.2**) is eliminated and replaced by a single-unit delay element.
- A 1-bit quantizer with two levels is used. The quantizer output simply indicates whether the present input sample $x(kT_s)$ is more or less compared to its accumulated approximation $\hat{x}(kT_s)$.
- Output $\hat{x}(kT_s)$ of the delay unit changes in small steps.
- The accumulator unit goes on adding the quantizer output with the previous accumulated version $\hat{x}(kT_s)$.
- $u(kT_s)$, is an approximate version of $x(kT_s)$.
- Performance of the Delta Modulation scheme is dependent on the sampling rate. Most of the above comments are acceptable only when two consecutive input samples are very close to each other.

Now, referring back to Fig. 3.14.2, we see that,

$$e(kT_s) = x(kT_s) - \{\hat{x}([k-1]T_s) + v([k-1]T_s)\}$$
3.14.3

Further,

$$v(kT_s) = e_q(kT_s) = s.sign[e(kT_s)]$$
3.14.4

Here, 's' is half of the step-size δ as indicated in **Fig. 3.14.3**.



Fig. 3.14.3 This diagram indicates the output levels of 1-bit quantizer. Note that if δ is the step size, the two output levels are $\pm s$

Now, assuming zero initial condition of the accumulator, it is easy to see that

$$u(kT_s) = s \sum_{j=1}^{k} sign[e(jT_s)]$$
$$u(kT_s) = \sum_{j=1}^{k} v(jT_s)$$
3.14.5

Further,

1.

$$\hat{x}(kT_s) = u([k-1]T_s) = \sum_{j=1}^{k-1} v(jT_s)$$
3.14.6

Eq. 3.14.6 shows that $\hat{x}(kT_s)$ is essentially an accumulated version of the quantizer output for the error signal $e(kT_s)$. $\hat{x}(kT_s)$ also gives a clue to the demodulator structure for DM. **Fig. 3.14.4** shows a scheme for demodulation. The input to the demodulator is a binary sequence and the demodulator normally starts with no prior information about the incoming sequence.



Fig. 3.14.4 Demodulator structure for DM

Now, let us recollect from our discussion on DPCM in the previous lesson (Eq. 3.13.3) that, $u(kT_s)$ closely represents the input signal with small quantization error $q(kT_s)$, i.e.

$$u(kT_s) = x(kT_s) + q(kT_s)$$
 3.14.7

Next, from the close loop including the delay-element in the accumulation unit in the Delta modulator structure, we can write

$$u([k-1]T_s) = \hat{x}(kTs) = x(kT_s) - e(kT_s) = x([k-1]T_s) + q([k-1]T_s)$$
3.14.8

Hence, we may express the error signal as,

$$e(kT_s) = \{x(kT_s) - x([k-1]T_s)\} - q([k-1]T_s)$$
3.14.9

That is, the error signal is the difference of two consecutive samples at the input except the quantization error (when quantization error is small).

Advantages of a Delta Modulator over DPCM

- a) As one sample of $x(kT_s)$ is represented by only one bit after delta modulation, no elaborate word-level synchronization is necessary at the input of the demodulator. This reduces hardware complexity compared to a PCM or DPCM demodulator. Bit-timing synchronization is, however, necessary if the demodulator in implemented digitally.
- b) Overall complexity of a delta modulator-demodulator is less compared to DPCM as the predictor unit is absent in DM.

However DM also suffers from a few limitations such as the following:

a) Slope over load distortion: If the input signal amplitude changes fast, the stepby-step accumulation process may not catch up with the rate of change (see the sketch in **Fig. 3.14.5**). This happens initially when the demodulator starts operation from cold-start but is usually of negligible effect for speech. However, if this phenomenon occurs frequently (which indirectly implies smaller value of auto-correlation co-efficient $R_{xx}(\tau)$ over a short time interval) the quality of the received signal suffers. The received signal is said to suffer from slope-overload distortion. An intuitive remedy for this problem is to increase the step-size δ but that approach has another serious lacuna as noted in b).



Fig. 3.14.5 A sketch indicating slope-overload problem. The horizontal axis represents time. The continuous line represents the analog input signal, before sampling and the stair-case represents the output $\hat{x}(kT_s)$ of the delay element.

b) Granular noise: If the step-size is made arbitrarily large to avoid slope-overload distortion, it may lead to 'granular noise'. Imagine that the input speech signal is fluctuating but very close to zero over limited time duration. This may happen due to pauses between sentences or else. During such moments, our delta modulator is likely to produce a fairly long sequence of 101010...., reflecting that the accumulator output is close but alternating around the input signal. This phenomenon is manifested at the output of the delta demodulator as a small but perceptible noisy background. This is known as 'granular noise'. An expert listener can recognize the crackling sound. This noise should be kept well within a tolerable limit while deciding the step-size. Larger step-size increases the granular noise while smaller step size increases the degree of slope-overload distortion. In the first level of design, more care is given to avoid the slope-overload distortion. We will briefly discuss about this approach while keeping the step-size fixed. A more efficient approach of adapting the step-size, leading to Adaptive Delta Modulation (ADM), is excluded.

Condition for avoiding slope overload: From **Fig. 3.14.3** we may observe that if an input signal changes more than half of the step size (i.e. by 's') within a sampling interval, there will be slope-overload distortion. So, the desired limiting condition on the input signal x(t) for avoiding slope-overloading is,

$$\frac{dx(t)}{dt}\Big|_{\max} \le \frac{s}{T_s}$$
3.14.10

Quantization Noise Power

Let us consider a sinusoid representing a narrow band signal $x(t) = a_m \cos(2\pi ft)$ where 'f' represents the maximum frequency of the signal and ' a_m ' its peak amplitude. There will be no slope-overload error if

$$\frac{s}{T_s} \ge 2\pi a_m f$$
 or $a_m \le \frac{s}{2\pi f T_s}$

The above condition effectively limits the power of x(t). The maximum allowable power

of x(t) = P_{max} =
$$\frac{a_m^2}{2} = \frac{s^2}{8\pi^2 f^2 T_s^2}$$

Once the slope overload distortion has been taken care of, one can find an estimate of $SQNR_{max}$. Assuming uniform quantization noise between +s and -s, the quantization noise power is

$$N_Q = \frac{4s^2}{12} = \frac{s^2}{3}$$

Let us now recollect that the sampling frequency $f_s = 1/T_s$ is much greater than 'f'. The granular noise due to the quantizer can be approximated to be of uniform power spectral density over a frequency band upto f_s (**Fig. 3.14.6**). The low pass filter at the output end of the delta demodulator is designed as per the bandwidth of x(t) and much of the quantization noise power is filtered off. Hence, we may write,

the in-band quantization noise power
$$\approx \frac{f}{f_s} N_Q$$

Therefore, SQNR_{max}= (Maximum signal power) / (In-band quantization noise power)

$$=(\frac{3}{8\pi^2}).(\frac{f_s}{f})^3$$

The above expression indicates that we can expect an improvement of about 9dB by doubling the sampling rate and it is not a very impressive feature when compared with a PCM scheme. Typically, when the permissible data rate after quantization and coding of speech signal is more than 48 Kbps, PCM offers better SQNR compared to linear DM.



Fig. 3.14.6 In-band noise power at the output of the low pass filter in a delta demodulator is shown by the shaded region

Problems

- Q3.14.1) Comment if a delta modulator can also be called as a 1-bit DPCM scheme.
- Q3.14.2) Mention two differences between DPCM and Delta Modulator
- Q3.14.3) Suggest a solution for controlling the granular noise at the output of a delta modulator.
- Q3.14.4) Let $x(t) = 2 \cos (2\pi \times 100t)$. If this signal is sampled at 1 KHz for delta modulator, what is the maximum achievable SQNR in dB?