# 4 Compressible Fluid Dynamics

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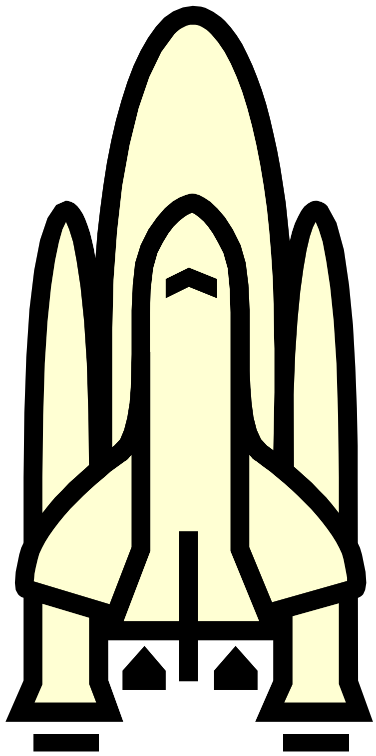


## 4.1 Compressible flow definitions

Compressible flow describes the behaviour of fluids that experience significant variations in density under the application of external pressures. For flows in which the density does not vary significantly, the analysis of the behaviour of such flows may be simplified greatly by assuming a constant density and the fluid is termed incompressible. This is an idealisation, which leads to the theory of incompressible flow. However, in the many cases dealing with gases (especially at higher velocities) and those cases dealing with liquids with large pressure changes, significant variations in density can occur, and the flow should be analysed as a compressible flow if accurate results are to be obtained.

Allowing for a change in density brings an additional variable into the analysis. In contrast to incompressible flows, which can usually be solved by considering only conservation of mass and conservation of momentum. Usually, the principle of conservation of energy is included. However, this introduces another variable (temperature), and so a fourth equation (such as the ideal gas equation) is required to relate the temperature to the other thermodynamic properties in order to fully describe the flow.

**Fundamental assumptions**

1. The gas is continuous.
2. The gas is perfect (obeys the perfect gas law)

3. Gravitational effects on the flow field are negligible.
4. Magnetic and electrical effects are negligible.
5. The effects of viscosity are negligible.

**Applied principles**

1. Conservation of mass (continuity equation)
2. Conservation of momentum (Newton's law)
3. Conservation of energy (first law of thermodynamics)
4. Equation of state

## 4.2 Derivation of the Speed of sound in fluids

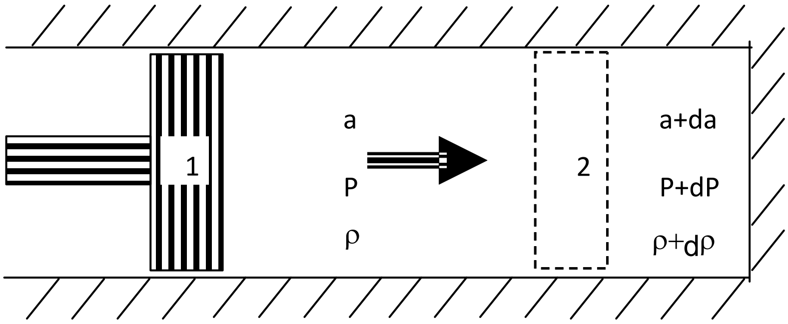



**Figure 4.1** Propagation of sound waves through a fluid

Consider the control volume surrounding the cylinder and its content in Figure 4.1, conservation of mass between the sides of the piston at section 2 implies:

$$\rho.A.a = (\rho+\delta\rho).A.(a+da)$$

Since "A" is area of cross-section of the piston (constant); "ρ "is the density of the fluid and "a" is the speed of sound wave propagated through the fluid.

Expand the above to get

$$(\rho.da + a.d\rho) = 0 \quad (4.1)$$

Applying the Momentum Equation to the same section:

$P.A - (P + dP).A = \rho.A.a\ (a+da-a\ )$
Hence $dP = -\ \rho.a.da$
but $\rho da = -\ a.d\rho$
ie $dP = -\ a.(-\ a.d\rho)$

Hence $$a = \sqrt{\frac{dP}{d\rho}} \quad (4.2)$$

This is the expression for the speed of sound.

**The Speed of sound for liquids**

In order to evaluate the speed of sound for liquids, the bulk modulus of elasticity relating the changes in density of the fluid due to the applied pressure in equation 4.2:

$$K_s = \frac{dP}{d\rho/\rho}$$

*rewrite*

$$\frac{dP}{d\rho} = \frac{K_s}{\rho}$$

*hence*

$$a = \sqrt{\frac{dP}{d\rho}} = \sqrt{\frac{K_s}{\rho}} \quad (4.3)$$

**The speed of sound for an ideal gas**

Starting from equation 4.2

$$a = \sqrt{\frac{dP}{d\rho}}$$

$$a^2 = \frac{dP}{d\rho} = \frac{P.dP/P}{d\rho} = \frac{P.d(\ln P)}{d\rho}$$

Since for a perfect gas $P = C.\rho^{\gamma}$

Then

$$a^2 = \frac{P.d(\ln(C.\rho^{\gamma}))}{d\rho} = \frac{P.\gamma.C.\rho^{\gamma-1}}{C.\rho^{\gamma}} = \frac{\gamma.P}{\rho}$$

Hence $a = \sqrt{\gamma.R.T}$ (4.4)

Maxwell was the first to derive the speed of sound for gas.

| Gas | Speed (m/s) |
|---|---|
| Air | 331 |
| Carbon Dioxide | 259 |
| Oxygen | 316 |
| Helium | 965 |
| Hydrogen | 1290 |

**Table 4.1** The speed of sound for various gases at 0° C

## 4.3 The Mach number

Mach number is the ratio of the velocity of a fluid to the velocity of sound in that fluid, named after Ernst Mach (1838-1916), an Austrian physicist and philosopher. In the case of an object moving through a fluid, such as an aircraft in flight, the Mach number is equal to the velocity of the airplane relative to the air divided by the velocity of sound in air at that altitude. Mach numbers less than one indicate subsonic flow; those greater than one, supersonic flow. The Mach number can be expressed as

$$M = V / a \tag{4.5}$$

*Where*

*M = Mach number*
*V = fluid flow velocity (m/s)*
*a = speed of sound (m/s)*

Alternatively the Mach number can be expressed with the density and the bulk modulus for elasticity as

$$M = V\,(\rho / K_s)^{1/2} \qquad (4.6)$$

*Where*

$\rho$ = *density of fluid (kg/m³)*
$K_s$ = *bulk modulus elasticity (N/m² (Pa))*

The bulk modulus elasticity has the dimension pressure and is commonly used to characterize the fluid compressibility. The square of the Mach number is the Cauchy Number. ( C )

$$M^2 = C \qquad (4.7)$$

As the aircraft moves through the air it makes pressure waves. These pressure waves stream out away from the aircraft at the speed of sound. This wave acts just like the ripples through water after a stone is dropped in the middle of a still pond. At Mach 1 or during transonic speed (Mach 0.7 - 0.9), the aircraft actually catches up with its own pressure waves. These pressure waves turn into one big shock wave. It is this shock wave that buffets the airplane. The shock wave also creates high drag on the airplane and slows the airplane's speed. As the airplane passes through the shock wave it is moving faster than the sound it makes. The shock wave forms an invisible cone of sound that stretches out toward the ground. When the shock wave hits the ground it causes a sonic boom that sounds like a loud thunderclap.

The energy lost in the process of compressing the airflow through these shock waves is called wave drag. This reduces lift on the airplane.

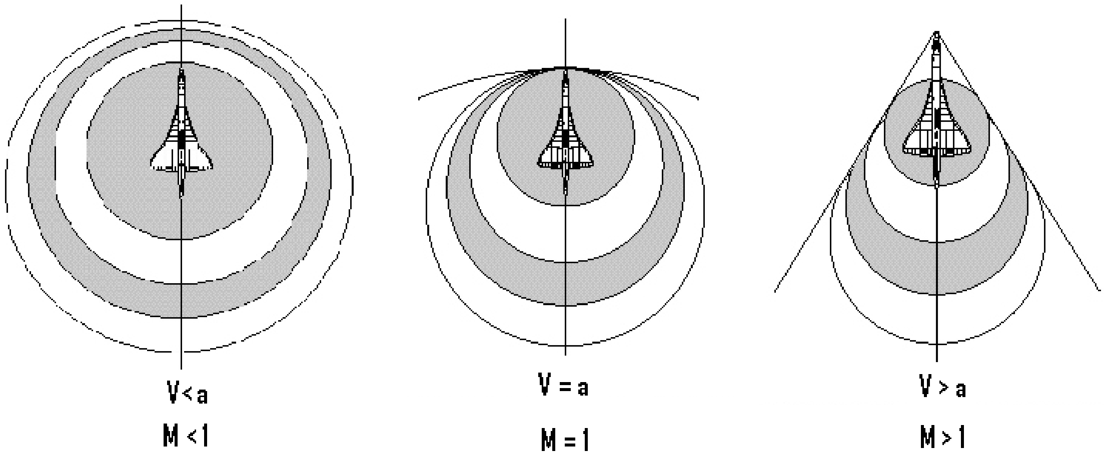



**Figure 4.2** Propagation of sound waves through a fluid

**Mach number and flow regimes:**

Mach number represents the ratio of the speed of an object such as aeroplane in air, or the relative motion of air against the aeroplane. It is commonly agreed that for Mach numbers less than 0.3, the fluid is considered incompressible. The following zoning based on the value of Mach numbers are universally agreed.

Ma < 0.3; incompressible flow
0.3 < Ma < 0.8; subsonic flow, no shock waves
0.8 < Ma < 1.2; transonic flow, shock waves
1.2 < Ma < 5.0; supersonic flow
5 < Ma; hypersonic flow

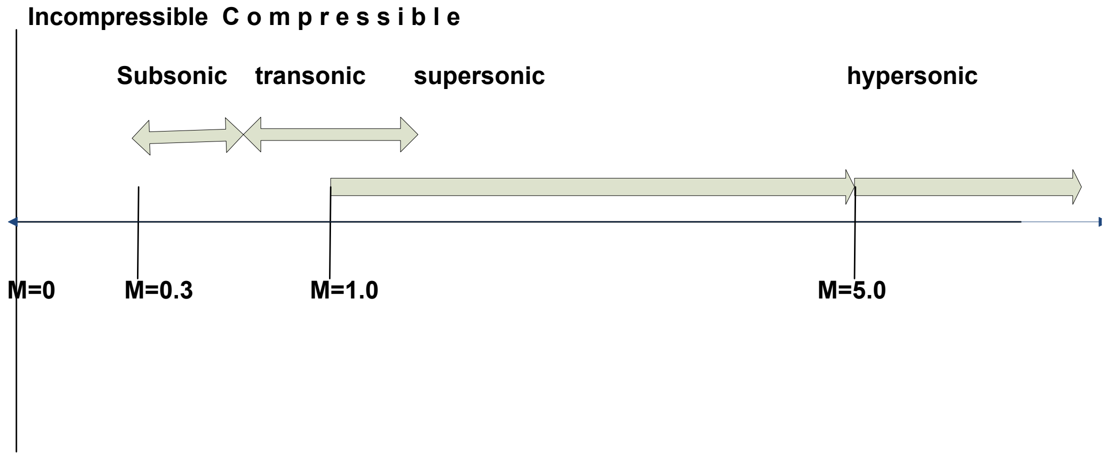



**Figure 4.3** Compressible flow regimes

## 4.4 Compressibility Factor

For a compressible fluid the energy equation between two sections 1 and 2 is represented by Bernoulli's theorem:

$$\int \frac{dP}{\rho g} + \frac{V^2}{2g} + z = C$$
$$gas\ equation: P = k.\rho^{\gamma}$$
$$hence\ \rho = (P/k)^{1/\gamma}$$
$$\int \frac{1}{g}\frac{dP}{(P/k)^{1/\gamma}} + \frac{V^2}{2g} + z = C$$

$$\frac{\gamma}{\gamma - 1}[\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}] + \frac{V_2^2}{2g} + z_2 - \frac{V_1^2}{2g} - z_1 = 0 \tag{4.8}$$

In cases where the fluid comes to rest, $V_2$ =0, and if the stream line is horizontal, the z-terms cancel out, hence the above equation reduces to

$$\frac{\gamma}{\gamma - 1}[\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}] - \frac{V_1^2}{2g} = 0 \tag{4.9}$$

Since $P = k.\rho^{\gamma}$ hence

$$\frac{\rho_1}{\rho_2} = (\frac{P_1}{P_2})^{1/\gamma}$$
$$and$$
$$\frac{P_1}{\rho_1} = R.T_1 = \frac{\gamma.R.T_1}{\gamma} = \frac{a^2}{\gamma} = \frac{V_1^2}{\gamma.M^2}$$

Hence equation 4.9 can be written in terms of the final pressure as

$$\frac{\gamma}{\gamma-1}.\frac{P_1}{\rho_1}[\frac{P_2}{\rho_2}/\frac{P_1}{\rho_1}-1]-\frac{V_1^{\,2}}{2}=0$$

$$\frac{\gamma}{\gamma-1}.\frac{V_1^{\,2}}{\gamma.M^2}[\frac{P_2}{P_1}x(\frac{P_1}{P_2})^{1/\gamma}-1]-\frac{V_1^{\,2}}{2}=0$$

$$\frac{1}{\gamma-1}.\frac{1}{.M^2}[\frac{P_2}{P_1})^{1-1/\gamma}-1]-\frac{1}{2}=0$$

Hence

$$\frac{P_2}{P_1}=(1+\frac{\gamma-1}{2}M^2)^{\frac{\gamma}{\gamma-1}} \tag{4.10}$$

Equation (4.10) can be expanded as follows:

$$\frac{P_2}{P_1}=1+\frac{\gamma}{2}M^2+\frac{\gamma}{8}M^4+\frac{\gamma(2-\gamma)}{48}M^6+......etc$$

$$or$$

$$P_2-P_1=P_1[\frac{\gamma}{2}M^2.+\frac{\gamma}{8}M^4+\frac{\gamma(2-\gamma)}{48}M^6+......etc]$$

$$but$$

$$M^2=\frac{V_1^{\,2}}{a^2}=\frac{V_1^{\,2}}{\gamma.P_1/\rho_1}$$

$$P_1=\frac{\rho_1.V_1^{\,2}}{\gamma.M^2}$$

$$hence$$

$$P_2-P_1=\frac{\rho_1.V_1^{\,2}}{2}[1+\frac{1}{4}M^2+\frac{(2-\gamma)}{24}M^4+......etc]$$

$$hence$$

$$CF=1+\frac{1}{4}M^2+\frac{(2-\gamma)}{24}M^4+.... \tag{4.11}$$

CF is the compressibility factor.

Comparison between Incompressible and Compressible fluid flow of gases. In terms of the velocity of flow, the expression for a compressible fluid is given by equation 4.8

$$\frac{\gamma}{\gamma-1}[\frac{P_2}{\rho_2}-\frac{P_1}{\rho_1}]+\frac{V_2^{\ 2}}{2}+gz_2-\frac{V_1^{\ 2}}{2}-gz_1=0$$

The incompressible situation, Bernoulli's equation is given by

$$[\frac{P_2}{\rho_2}-\frac{P_1}{\rho_1}]+\frac{V_2^{\ 2}}{2}+gz_2-\frac{V_1^{\ 2}}{2}-gz_1=0$$

It is obvious that the term (γ/γ-1) is the difference, for air the value of this term is 3.5, affecting the pressure head term, velocity term and elevation terms are not affected by this term.

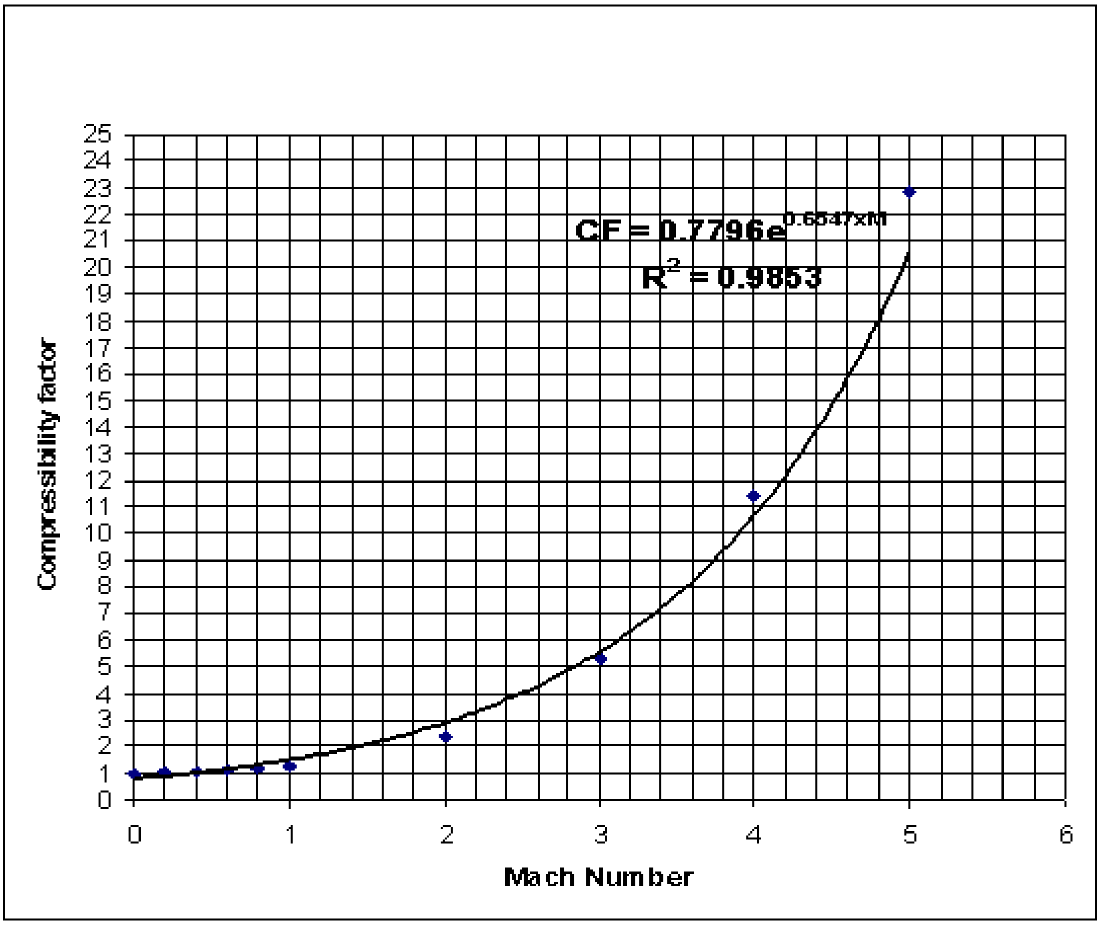



**Figure 4.4** Compressibility Factor

## 4.5 Energy equation for frictionless adiabatic gas processes

Consider a one-dimensional flow through a duct of variable area, the Steady Flow energy Equation between two sections 1 and 2:

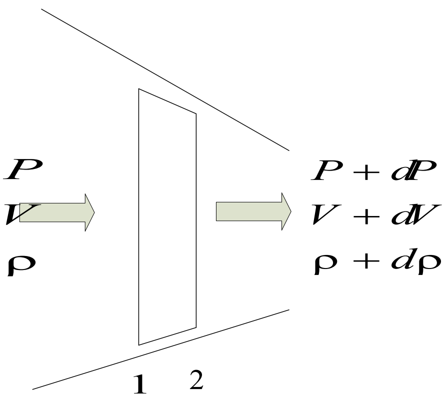



**Figure 4.5** One dimensional compressible flow

$$Q - W = m\,[\,(h_2 - h_1) + (V_2^2 - V_1^2)/2 + g(z_2 - z_1)]$$

If the flow is adiabatic, and there is no shaft work and assume horizontal duct, the equation reduces to

$$(h_2 - h_1) + (V_2^2 - V_1^2)/2 = 0 \qquad (4.12)$$

Or in general $h + V^2/2 = \text{constant}$

By differentiation $dh + v\,dv = 0$

But the first law of thermodynamics states that
$dQ - dW = du$

The second law of thermodynamics states that $dQ = T.dS$

Also $dW = P.\,d(1/\rho)$
and $h = u + P/\rho$ or $du = dh - P.\,d(1/\rho) + (1/\rho).dP$

Hence the 1st law of thermodynamics is written as

$T.dS = dh - P.\,d(1/\rho) - (1/\rho).dP + P.\,d(1/\rho)$
$T.dS = dh - dP/\rho$

For isentropic process, $dS = 0$
Then $dh = dP/\rho$

but $dh = -v.dv$
hence $-v.\,dv = dP/\rho$
Therefore $dP/dv = -\rho\, v$ (4.13)

The continuity equation states that $\rho\, A\, v = \text{constant}$

So by differentiation

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

*hence*

$$\frac{dA}{A} = -\frac{dV}{V} - \frac{d\rho}{\rho}$$

*but*

$$\frac{dV}{V} = -\frac{dP}{\rho.V^2}$$

*hence*

$$\frac{dA}{A} = \frac{dP}{\rho.V^2} - \frac{d\rho}{\rho}$$

since $a^2 = dP/d\rho$

*the*

$$\frac{dA}{A} = \frac{dP}{\rho.V^2}(1 - \frac{V^2}{a^2}) = \frac{dP}{\rho.V^2}(1 - M^2)$$

*finally*

$$\frac{dA}{dP} = \frac{A}{\rho.V^2}(1 - M^2) \tag{4.14}$$

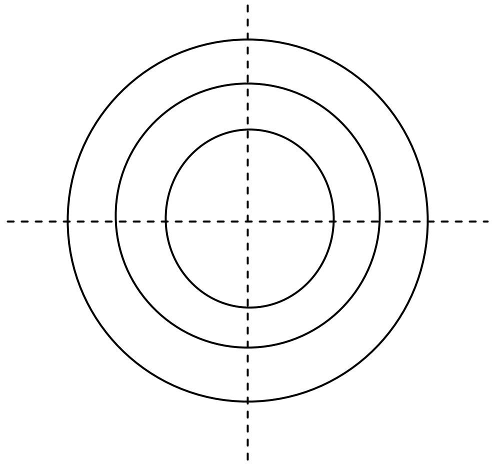
Similarly it is possible to show that

$$\frac{dA}{dV} = \frac{A}{V}(1 - M^2) \tag{4.15}$$

To illustrate the above relationships between changes in area of duct and the changes in velocity and pressure, figure 4.6 is drawn.

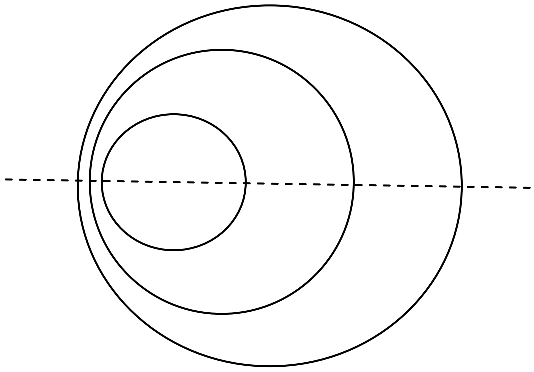
**Case 1**

$$\frac{dA}{dV} > 0$$
*and*
$$\frac{dA}{dV} < 0$$



**Case 2**

$$\frac{dA}{dV} = 0$$
*and*
$$\frac{dA}{dV} = 0$$



**Case 3**

$$\frac{dA}{dV} < 0$$
*and*
$$\frac{dA}{dV} > 0$$

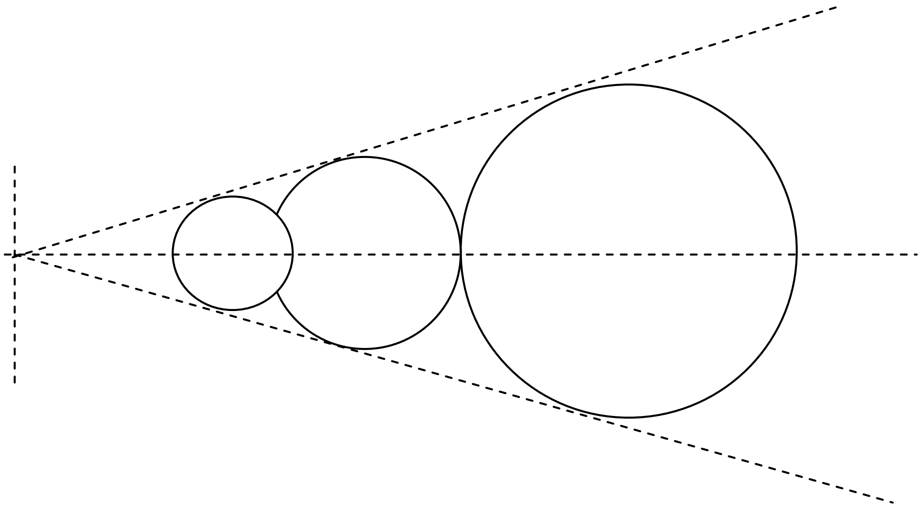


**Figure 4.6** Changes of area and its effect on pressure and velocity of compressible flow

## 4.6 Stagnation properties of compressible flow

Stagnation condition refers to the situation or rather position in which the fluid becomes motionless. There are many examples of this in real applications; two are shown in figure 4.7

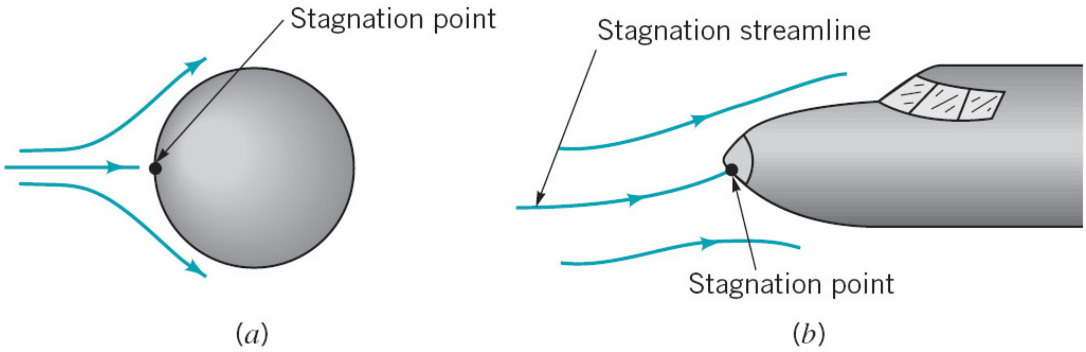



**Figure 4.7** Stagnation situations in flow applications

When defining what is meant by a compressible flow, it is useful to compare the density to a reference value, such as the stagnation density, $\rho_0$, which is the density of the fluid if it were to be slowed down isentropically to stationary.

Recall the simplified energy equation for the duct in the previous section, between any section, and rest (stagnation).

$$h + V^2/2 = h_o$$

The enthalpy is defined as the product of the specific heat capacity Cp and the temperature of the fluid, T. also note that

$$Cp = \frac{\gamma . R}{\gamma - 1}$$

Hence, the energy equation can be written as:

$$\frac{a^2}{\gamma - 1} + \frac{V^2}{2} = \frac{{a_o}^2}{\gamma - 1}$$

Since M = V/a then

$$\frac{a^2}{\gamma - 1} + \frac{a^2 . M^2}{2} = \frac{{a_o}^2}{\gamma - 1}$$

$$a^2 (1 + \frac{\gamma - 1}{2} M^2) = {a_o}^2 \qquad (4.16)$$

Plotting the speed of sound ratio ($a/a_o$) versus M, is shown in Figure 4.8

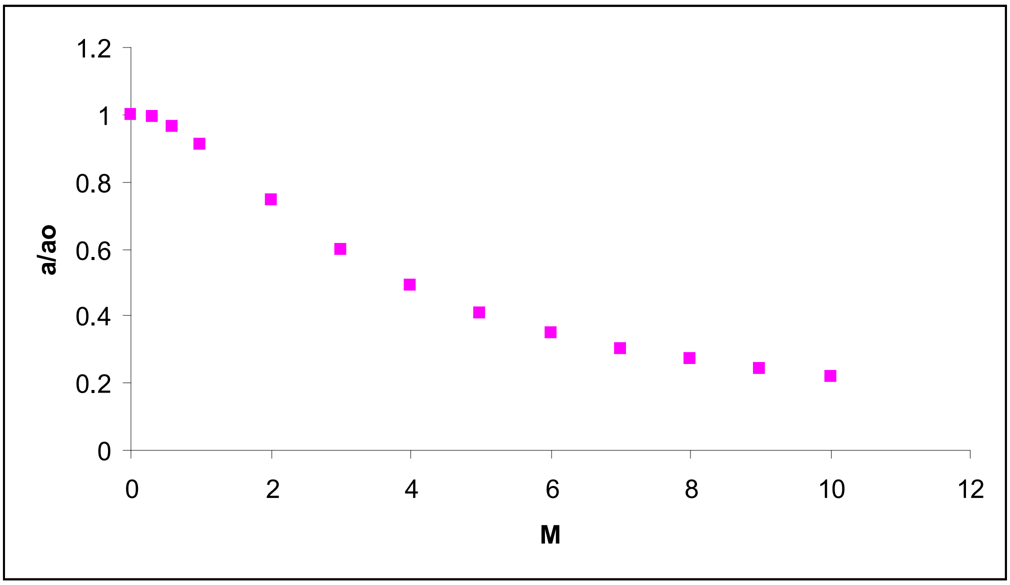



**Figure 4.8** Variation of speed of sound ratio with Mach number

Recall the energy equation for a fluid with a stagnation state "o"

$$h + V^2/2 = h_o$$

Use h = Cp.T, the energy equation can be written as:

$$Cp.T + \frac{V^2}{2} = Cp.T_o$$
$$hence$$
$$\frac{T_o}{T} = 1 + \frac{V^2}{2.Cp.T}$$
$$but$$
$$V^2 = a^2.M^2 \cdots\cdots and \cdots Cp = \frac{\gamma.R}{\gamma - 1}$$
$$\therefore \frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 \qquad (4.17)$$

In order to find the maximum velocity for stagnation condition, the EE is used With velocity being maximum when T is taken down to absolute zero, ie

$$Cp.(T = 0) + \frac{V^2}{2} = Cp.T_o$$
$$hence$$
$$V_{\max} = \sqrt{2.Cp.T_o} \qquad (4.18)$$

Other Stagnation relationships

Starting with the stagnation temperature ratio, it is possible to derive a similar relationship for stagnation pressure ratio

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$
$$but$$
$$\frac{P_o}{P} = [\frac{T_o}{T}]^{\frac{\gamma}{\gamma - 1}}$$
$$hence$$

$$\frac{P_o}{P} = [1 + \frac{\gamma - 1}{2} M^2]^{\frac{\gamma}{\gamma - 1}} \qquad (4.19)$$

For the stagnation density ratio

$$with\ \frac{P}{\rho^{\gamma}} = c$$

$$\frac{\rho_o}{\rho} = [\frac{P_o}{P}]^{\frac{1}{\gamma}} = [1 + \frac{\gamma - 1}{2} M^2]^{\frac{1}{\gamma - 1}} \tag{4.20}$$

## 4.7 Worked Examples

### Worked Example 4.1

Calculate the speed of sound in air and in water at *0 °C and at 20 °C* and absolute pressure *1 bar*.
For air - γ = 1.4 and R = 287 (J/K kg)
For water $K_s$= 2.06 $10^9$ (N/$m^2$) and ρ = 998 (kg/$m^3$) at 0 °C, and 1000 (kg/$m^3$) at 20 °C

**Solution:**

For air at 0 °C

a= $[\gamma\ R\ T]^{1/2}$ = $(1.4\ (287\ J/K\ kg)\ (273\ K))^{1/2}$ = 331.2 (m/s)
Where γ = 1.4 and R = 287 (J/K kg)

The speed of sound in air at 20 °C and absolute pressure 1 bar can be calculated as
a = $[\gamma\ R\ T]^{1/2}$ = $(1.4\ (287\ J/K\ kg)\ (293\ K))^{1/2}$ = 343.1 (m/s)
The difference is = 3.6%

The speed of sound in water at 0 °C can be calculated as

$$a = \sqrt{\frac{K_s}{\rho}} = \sqrt{\frac{2.06x10^9}{998}} = 1437\ m/s$$

Where $K_s$= 2.06 $10^9$ (N/$m^2$) and ρ = 998 (kg/$m^3$)

The speed of sound in water at 20 °C can be calculated as

$$a = \sqrt{\frac{K_s}{\rho}} = \sqrt{\frac{2.06x10^9}{1000}} = 1445\ m/s$$

Where $K_s$= 2.06 $10^9$ (N/$m^2$) and ρ = 1000 (kg/$m^3$)
The difference is = 0.5%

It can be noted that the speed of sound in gases changes more than in liquids with changes in temperature.

## Worked Example 4.2

An aircraft flies at an altitude of 10,000 m where the pressure and density are 0.265 bar and 0.41 kg/m$^3$ respectively.

a) Determine the aircraft speed if the Mach number is 1.5
b) What is the speed of the plane at sea level if the Mach number is maintained?

**Solution:**

a) The speed of sound in air is calculated first, then using the Mach definition, the speed of the aircraft is calculated as follows:

$$a = \sqrt{\gamma.R.T}$$

$$a = \sqrt{\gamma.P/\rho} = \sqrt{1.4x0.265x10^5/0.41} = 300.8\ m/s$$

$$V = a.M = 300.8x1.5 = 451\ m/s$$

b) when the Mach number is M = 1.5, similar method to that in (a) is used:

$$a = \sqrt{\gamma.R.T}$$

$$a = \sqrt{\gamma.P/\rho} = \sqrt{1.4x1.01325x10^5/1.2} = 343.8\ m/s$$

$$V = a.M = 343.8x1.5 = 515.7\ m/s$$

## Worked Example 4.3

A sealed tank filled with air which is maintained at 0.37 bar gauge and 18°C. The air discharges to the atmosphere (1.013 bar) through a small opening at the side of the tank.

a) Calculate the velocity of air leaving the tank; assume the flow to be compressible and the process to be frictionless adiabatic.
b) Compare the value if the flow is incompressible.
c) comment on the result.

Take for air, R=287 J/kgK, and γ= 1.4.

**Solution:**

a) Bernoulli equation for a compressible case, Assume $z_2=z_1$ and $V_1 = 0$; The equation reduces to:

$$\frac{V_2^{\ 2}}{2} = \frac{\gamma}{\gamma - 1}[\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}]$$

$$\therefore V_2 = \left[\frac{2\gamma}{\gamma-1}[\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}]\right]^{0\ 5} = \left[\frac{2\gamma}{\gamma-1} x \frac{P_1}{\rho_1}[1-(\frac{P_2}{P_1})^{\frac{\gamma-1}{\gamma}}]\right]^{0\ 5}$$

Since $\frac{P_1}{\rho_1} = R.T_1$ Then the discharge velocity is:

$$\therefore V_2 = \left[\frac{2\gamma}{\gamma-1} xR.T_1[1-(\frac{P_2}{P_1})^{\frac{\gamma-1}{\gamma}}]\right]^{0.5} = \left[\frac{2x1.4}{1.4-1} x287x291[1-(\frac{1.013}{1.013+0.37})^{\frac{1.4-1}{1.4}}]\right]^{0.5} = 223\ m/s$$

b) For incompressible fluids $[\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}] + \frac{V_2^{\ 2}}{2} + gz_2 - \frac{V_1^{\ 2}}{2} - gz_1 = 0$

With $\rho_2 = \rho_1$ and again with $z_2$=$z_1$ and $V_1$ = 0; The equation reduces to:

$$\therefore V_2 = \left[\frac{2(P_1 - P_2)}{\rho}\right]^{0.5} = \left[\frac{2x0.37}{\frac{(0.37+1.013)x10^5}{287x291}}\right]^{0.5} = 211\ m/s$$

c) The fluid velocity is different for the two assumptions,

$$a = \sqrt{\gamma.R.T} = \sqrt{1.4x287x291} = 342\ m/s$$
$$hence$$
$$M = \frac{v}{a} = \frac{223}{342} = 0.6$$

The fluid is clearly compressible, so the accurate velocity is 223 m/s

## Worked Example 4.4

A low flying missile develops a nose temperature of 2500K where the temperature and pressure of the atmosphere at that elevation are 0.03bar and 220K respectively. Determine the missile velocity and the stagnation pressure. Assume for air Cp=1000 J/kgK and γ=1.4.

**Solution:**

Using the stagnation relations,

$$Cp = Cv + R$$
$$Cp = Cp/\gamma + R$$
$$hence$$
$$Cp = \frac{\gamma.R}{\gamma - 1}$$

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$M = V/a = V/\sqrt{\gamma.R.T}$$

$$hence$$
$$\frac{T_o}{T} = 1 + \frac{V^2}{2.Cp.T}$$

$$V = \sqrt{2xCpxTx\left(\frac{T_o}{T} - 1\right)}$$

$$V = 2135\ m/s$$

Similarly the stagnation density ratio can be used to determine the stagnation pressure:

$$\frac{P_o}{P} = [1 + \frac{V^2}{2.Cp.T}]^{\frac{\gamma}{\gamma-1}}$$

*hence*

$$P_o = 0.03x[1 + \frac{2135^2}{2x1000x220}]^{\frac{1.4}{0.4}} = 148\ bar$$

## Worked Example 4.5

An air stream at 1 bar, 400 K moving at a speed of 400 m/s is suddenly brought to rest. Determine the final pressure, temperature and density if the process is adiabatic.

Assume for air: γ = 1.4. Cp = 1005 J/kgK and density = 1.2 kg/m$^3$.

**Solution:**

Using the stagnation relations,

$$T_o = T + \frac{V^2}{2.Cp} = 400 + \frac{400^2}{2x1005}$$

$$= 479.5\,K$$

$$\frac{P_o}{P} = [1 + \frac{V^2}{2.Cp.T}]^{\frac{\gamma}{\gamma-1}}$$

*hence*

$$P_o = 1x[1 + \frac{400^2}{2x1005x479.5}]^{\frac{1.4}{0.4}} = 1.711\,bar$$

$$\frac{\rho_o}{\rho} = [\frac{P_o}{P}]^{\frac{1}{\gamma}}$$

*hence*

$$\rho_o = 1.2x[\frac{1.711}{1}]^{\frac{1}{1.4}} = 1.761\,kg/m^3$$

## 4.8 Tutorial Problems - Compressible Flow

4.1 Assuming the ideal gas model holds, determine the velocity of sound in

a) air (*mwt* 28.96) at 25°C, with $\gamma = 1.4$,
b) argon (*mwt* 39.95) at 25°C, with $\gamma = 1.667$.

Ans[346 m/s, 321.5 m/s]

4.2 An airplane can fly at a speed of 800km/h at sea-level where the temperature is 15°C. lf the airplane flies at the same Mach number at an altitude where the temperature is -44°C, find the speed at which the airplane is flying at this altitude.

Ans[198 m/s]

4.3 A low flying missile develops a nose temperature of 2500K when the ambient temperature and pressure are 250K and 0.01 bar respectively. Determine the missile velocity and its stagnation pressure. Assume for air: $\gamma = 1.4$. Cp = 1005 J/kgK

Ans[2126 m/s, 31.6 bar ]

4.4 An airplane is flying at a relative speed of 200 m/s when the ambient air condition is 1.013 bar, 288 K. Determine the temperature, pressure and density at the nose of the airplane. Assume for air: $\gamma = 1.4$, density at ambient condition = 1.2 kg/m$^3$ and Cp =1005 J/kgK.

Ans[$T_o$=307.9K, $P_o$=1.28 bar, $\rho$ =1.42 kg/m$^3$ ]