Game Theory

Introduction

Game theory was developed for the purpose of analyzing competitive situations involving conflicting interests. In other words, game theory is used for decision making under conflicting situations where there are one or more opponents (i.e., players). For example, chess, poker, etc., are the games which have the characteristics of a competition and are played according to definite rules. Game theory provides solutions to such games, assuming that each of the players wants to maximize his profits and minimize his losses.

How to select the optimal strategy without knowledge of the competitors is the basic problem of playing a game?

The game theory models can be classified into several categories. Some important categories are listed below.

- **Two-person & N-person games:** If the number of players is two, it is known as two-person game. On the other hand, if the number of players is N, it is known as N-person game.
- **Zero sum & Non-zero sum game:** In a zero sum game, the sum of the points won equals the sum of the points lost, i.e., one player wins at the expense of the other. To the contrary, if the sum of gains or losses is not equal to zero, it is either positive or negative, then it is known as non-zero sum game. An example of non-zero sum game is the case of two competing firms each with a choice regarding its advertising campaign. In such a situation, both the firms may gain or loose, though their gain or loss may not be equal.
- **Games of Perfect and Imperfect information:** If the strategy of a player can be discovered by his competitor, then it is known as a perfect information game. In case of imperfect information games no player has complete information and tries to guess the real situation.
- **Pure & Mixed strategy games:** If the players select the same strategy each time, then it is referred to as pure strategy games. If a player decides to choose a course of action for each play in accordance with some particularly probability distribution, it is called mixed strategy game.

? What are the underlying assumptions of game theory?

- There are finite number of competitors (players).
- The players act reasonably.
- Every player strives to maximize gains and minimize losses.
- Each player has finite number of possible courses of action.
- The choices are assumed to be made simultaneously, so that no player knows his opponent's choice until he has decided his own course of action.
- The pay-off is fixed and predetermined.

- The pay-offs must represent utilities.

Basic Terminology

Player

Each participant (interested party) is called a player.

Strategy

The strategy of a player is the predetermined rule by which a player decides his course of action from the list of courses of action during the game. A strategy may be of two types:

- **Pure strategy.** It is a decision, in advance of all plays, always to choose a particular course of action. In other words, if the best strategy for each player is to play one particular strategy throughout the game, it is called pure strategy.
- **Mixed strategy**. It is a decision, in advance of all plays, to choose a course of action for each play in accordance with some particular probability distribution. In other words, if the optimal plan for each player is to employ different strategies at different times, we call it mixed strategy.

Optimal strategy

The course of action which maximizes the profit of a player or minimizes his loss is called an optimal strategy.

Saddle point

A saddle point is an element of the matrix that is both the smallest element in its row and the largest element in its column. Furthermore, saddle point is also regarded as an equilibrium point in the theory of games.

Pay-off

The outcome of playing the game is called pay-off.

Pay-off Matrix

It is a table showing the outcomes or payoffs of different strategies of the game.

Value of the Game

It refers to the expected outcome per play, when players follow their optimal strategy. It is generally denoted by V.

In the subsequent sections of this chapter, we provide several trivial illustrations. It should be noted that we make no pretense about the realism of these illustrations.

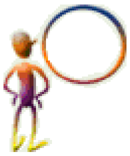
Pure Strategy

The simplest type of game is one where the best strategies for both players are pure strategies. This is the case if and only if, the pay-off matrix contains a saddle point. To illustrate, consider the following pay-off matrix concerning zero sum two person game.

Example

		Player B				
		I	**II**	**III**	**IV**	**V**
Player A	**I**	-2	0	0	5	3
	II	4	2	1	3	2
	III	-4	-3	0	-2	6
	IV	5	3	-4	2	-6

What is the optimal plan for both the players?



"The best plan is to profit by the folly of others." -*Pliny the Elder*

Solution.

We use the maximin (minimax) principle to analyze the game.

		Player B					
		I	**II**	**III**	**IV**	**V**	**Minimum**
Player A	**I**	-2	0	0	5	3	-2
	II	4	2	1	3	2	1
	III	-4	-3	0	-2	6	-4
	IV	5	3	-4	2	-6	-6
Maximum		5	3	1	5	6	

Select minimum from the maximum of columns.
Minimax = 1
Player A will choose II strategy, which yields the maximum payoff of 1.

Select maximum from the minimum of rows.
Maximin = 1
Similarly, player B will choose III strategy.

Since the value of maximin coincides with the value of the minimax, therefore, saddle point (equilibrium point) = 1.

The amount of payoff at an equilibrium point is also known as value of the game.

The optimal strategies for both players are: Player A must select II strategy and player B must select III strategy. The value of game is 1, which indicates that player A will gain 1 unit and player B will sacrifice 1 unit.

Mixed Strategy

In situations where a saddle point does not exist, the maximin (minimax) principle for solving a game problem breaks down. The concept is illustrated with the help of following example.

Example

Two companies A and B are competing for the same product. Their different strategies are given in the following pay-off matrix:

		Company B		
		I	**II**	**III**
Company A	**I**	-2	14	-2
	II	-5	-6	-4
	III	-6	20	-8

Determine the optimal strategies for both the companies.

Solution.

First, we apply the maximin (minimax) principle to analyze the game.

		Company B			
		I	**II**	**III**	**Minimum**
Company A	**I**	-2	14	-2	-2
	II	-5	-6	-4	-6
	III	-6	20	-8	-8
Maximum		-2	20	-2	

Minimax = -2
Maximin = -2

There are two elements whose value is –2. Hence, the solution to such a game is not unique.

In the above problem, there is no saddle point. In such cases, the maximin and minimax principle of solving a game problem can't be applied. Under this situation, both the companies may resort to what is known as mixed strategy.

In a mixed strategy, each player moves in a random fashion.

A mixed strategy game can be solved by following methods:

- Algebraic Method
- Calculus Method
- Linear Programming Method

Algebraic Method

Consider the zero sum two person game given below:

	Player B		
		I	**II**
Player A	**I**	a	b
	II	c	d

Formulas:

The solution of the game is:

A play's (p, 1 - p)

where:

$$p = \frac{d - c}{(a + d) - (b + c)}$$

B play's (q, 1 - q)

where:

$$q = \frac{d - b}{(a + d) - (b + c)}$$

$$\text{Value of the game, } V = \frac{ad - bc}{(a + d) - (b + c)}$$

Example 1

Consider the game of matching coins. Two players, A & B, put down a coin. If coins match (i.e., both are heads or both are tails) A gets rewarded, otherwise B. However, matching on heads gives a double premium. Obtain the best strategies for both players and the value of the game.

		Player B	
		I	**II**
Player A	**I**	2	-1
	II	-1	1

Solution.

This game has no **saddle point**.

$$p = \frac{1 - (-1)}{(2 + 1) - (-1 - 1)} = \frac{2}{5}$$

$1 – p = 3/5$

$$q = \frac{1 - (-1)}{(2 + 1) - (-1 - 1)} = \frac{2}{5}$$

$1 – q = 3/5$

$$V = \frac{2 X 1 - (-1) X (-1)}{(2 + 1) - (-1 - 1)} = \frac{1}{5}$$

Example 2

Solve the game whose payoff matrix is given below:

	Player B		
		I	**II**
Player A	**I**	1	7
	II	6	2

Solution.

This game has no **saddle point**.

$$p = \frac{2 - 6}{(1 + 2) - (7 + 6)} = \frac{2}{5}$$

$1 - p = 3/5$

$$q = \frac{2 - 7}{(1 + 2) - (7 + 6)} = \frac{1}{2}$$

$1 - q = 1/2$

$$V = \frac{1 X 2 - (7 X 6)}{(1 + 2) - (7 + 6)} = 4$$

Calculus Method

This method is almost similar to the previous method except that instead of equating the two expected values, the expected value for a given player is maximized.

Consider the zero sum two person game given below:

	Player B		
		I	**II**
Player A	**I**	a	b
	II	c	d

Formulas:

The solution of the game is:

A play's (p, 1 - p)

where:

$$p = \frac{d - c}{(a + d) - (b + c)}$$

B play's (q, 1 - q)

where:

$$q = \frac{d - b}{(a + d) - (b + c)}$$

Value of the game, $V = apq + c(1 - p)q + bp(1 - q) + d(1 - p)(1 - q)$

To illustrate this method, consider the same example discussed in the previous section.

Example 1

Consider the following game:

	Player B		
		I	**II**
Player A	**I**	2	-1
	II	-1	1

Solution.

This game has no **saddle point**.

$$p = \frac{1 - (-1)}{(2 + 1) - (-1 - 1)} = \frac{2}{5}$$

$1 – p = 3/5$

$$q = \frac{1 - (-1)}{(2 + 1) - (-1 - 1)} = \frac{2}{5}$$

$1 – q = 3/5$

V = 2 X 2/5 X 2/5 + (-1) X 3/5 X 2/5 + (-1) X 2/5 X 3/5 + 1 X 3/5 X 3/5 = 1/5

Example 2

Solve the game whose pay-off matrix is given below:

		Player B	
		I	II
Player A	I	1	3
	II	5	2

Solution.

This game has no **saddle point**.

$$p = \frac{2 - 5}{(1 + 2) - (3 + 5)} = \frac{3}{5}$$

1 – p = 2/5

$$q = \frac{2 - 3}{(1 + 2) - (3 + 5)} = \frac{1}{5}$$

1 – q = 4/5

V = 1 X 3/5 X 1/5 + 5 X 2/5 X 1/5 + 3 X 3/5 X 4/5 + 2 X 2/5 X 4/5 = 13/5

Linear Programming Method

The linear programming technique is used for solving mixed strategy games of dimensions greater than (2 X 2) size. The following simple example is used to explain the procedure.

Example

Two companies are competing for the same product. To improve its market share, company A decides to launch the following strategies.

- A1 = give discount coupons
- A2 = home delivery services
- A3 = free gifts

The company B decides to use media advertising to promote its product.

- B1 = internet
- B2 = newspaper
- B3 = magazine

	Company B			
		B1	**B2**	**B3**
Company A	**A1**	3	-4	2
	A2	1	-7	-3
	A3	-2	4	7

Use linear programming to determine the best strategies for both the companies.

Solution.

	Company B				**Minimum**
		B1	**B2**	**B3**	
Company A	**A1**	3	-4	2	-4
	A2	1	-7	-3	-7
	A3	-2	4	7	-2
Maximum		3	4	7	

Minimax = -2
Maximin = 3

This game has no **saddle point**. So the value of the game lies between –2 and +3. It is possible that the value of game may be negative or zero. Thus, a constant k is added to all the elements of pay-off matrix. Let k = 3, then the given pay-off matrix becomes:

	Company B			
Company A		**B1**	**B2**	**B3**
	A1	6	-1	5
	A2	4	-4	0
	A3	1	7	10

Let
V = value of the game
p1, p2 & p3 = probabilities of selecting strategies A1, A2 & A3 respectively.
q1, q2 & q3 = probabilities of selecting strategies B1, B2 & B3 respectively.

	Company B				**Probability**
Company A		**B1**	**B2**	**B3**	
	A1	6	-1	5	p1
	A2	4	-4	0	p2
	A3	1	7	10	p3
Probability		q1	q2	q3	

Company A's objective is to maximize the expected gains, which can be achieved by maximizing V, i.e., it might gain more than V if company B adopts a poor strategy. Hence, the expected gain for company A will be as follows:

$6p1 + 4p2 + p3 \geq V$
$-p1 - 4p2 + 7p3 \geq V$
$5p1 + 0p2 + 10p3 \geq V$
$p1 + p2 + p3 = 1$
and $p1, p2, p3 \geq 0$

Dividing the above constraints by V, we get
$6p1/V + 4p2/V + p3/V \geq 1$
$-p1/V - 4p2/V + 7p3/V \geq 1$
$5p1/V + 0p2/V + 10p3/V \geq 1$
$p1/V + p2/V + p3/V = 1/V$

To simplify the problem, we put
$p1/V = x_1$, $p2/V = x_2$, $p3/V = x_3$

In order to maximize V, company A can
Minimize $\mathbf{1/V = x_1 + x_2 + x_3}$
subject to

$6x_1 + 4x_2 + x_3 \geq 1$
$-x_1 - 4x_2 + 7x_3 \geq 1$
$5x_1 + 0x_2 + 10x_3 \geq 1$

and $x_1, x_2, x_3 \geq 0$

Company B's objective is to minimize its expected losses, which can be reduced by minimizing V, i.e., company A adopts a poor strategy. Hence, the expected loss for company B will be as follows:

$6q1 - q2 + 5q3 \leq V$
$4q1 - 4q2 + 0q3 \leq V$
$q1 + 7q2 + 10q3 \leq V$
$q1 + q2 + q3 = 1$

and $q1, q2, q3 \geq 0$

Dividing the above constraints by V, we get
$6q1/V - q2/V + 5q3/V \leq 1$
$4q1/V - 4q2/V + 0q3/V \leq 1$
$q1/V + 7q2/V + 10q3/V \leq 1$
$q1/V + q2/V + q3/V = 1/V$

To simplify the problem, we put
$q1/V = y_1$, $q2/V = y_2$, $q3/V = y_3$

In order to minimize V, company B can
Maximize $\mathbf{1/V = y_1 + y_2 + y_3}$
subject to

$6y_1 - y_2 + 5y_3 \leq 1$
$4y_1 - 4y_2 + 0y_3 \leq 1$
$y_1 + 7y_2 + 10y_3 \leq 1$

and $y_1, y_2, y_3 \geq 0$

Company B's problem is the dual of company A's problem.

To solve this problem, we introduce slack variables to convert inequalities to equalities. The problem becomes

Maximize $y_1 + y_2 + y_3 + 0y_4 + 0y_5 + 0y_6$

$6y_1 - y_2 + 5y_3 + y_4 = 1$
$4y_1 - 4y_2 + y_5 = 1$
$y_1 + 7y_2 + 10y_3 + y_6 = 1$

Initial Basic Feasible Solution

$y_1 = 0, y_2 = 0, y_3 = 0, z = 0$
$y_4 = 1, y_5 = 1, y_6 = 1$

Table 1

	c_j	1	1	1	0	0	0	
c_B	Basic Variables B	y_1	y_2	y_3	y_4	y_5	y_6	Solution values b (=X_B)
0	y_4	6	-1	5	1	0	0	1
0	y_5	4	-4	0	0	1	0	1
0	y_6	1	7	10	0	0	1	1
z_j-c_j		-1	-1	-1	0	0	0	

Key column = y_1 column
Minimum positive value = 1/6
Key row = y_4 row
Pivot element = 6
y_4 departs and y_1 enters.

Table 2

	c_j	1	1	1	0	0	0	
c_B	Basic Variable B	y_1	y_2	y_3	y_4	y_5	y_6	Solution values b (=X_B)
1	y_1	1	-1/6	5/6	1/6	0	0	1/6

0	y_5	0	-10/3	-10/3	-2/3	1	0	1/3
0	y_6	0	43/6	55/6	-1/6	0	1	5/6
z_j-c_j		0	-7/6	-1/6	1/6	0	0	

Final Table

	c_j	1	1	1	0	0	0	
c_B	**Basic Variable B**	y_1	y_2	y_3	y_4	y_5	y_6	**Solution values b (=X_B)**
1	y_1	1	0	45/43	7/43	0	1/43	8/43
0	y_5	0	0	40/43	-32/43	1	20/43	31/43
1	y_2	0	1	55/43	-1/43	0	6/43	5/43
z_j-c_j		0	0	57/43	6/43	0	7/43	

The values for y_1, y_2 & y_3 are 8/43, 5/43 & 0 respectively.

I/V = $y_1 + y_2 + y_3$ = 8/43 + 5/43 + 0 = 13/43
or V = 43/13

Company B's optimal strategy
q1 = V X y_1 = 43/13 X 8/43 = 8/13
q2 = V X y_2 = 43/13 X 5/43 = 5/13
q3 = V X y_3 = 43/13 X 0 = 0

Hence, company B's optimal strategy is (8/13, 5/13, 0).

Company A's optimal strategy

The values for x_1, x_2 & x_3 can be obtained from the final simplex table.

x_1 = 6/43, x_2 = 0 & x_3 = 7/43
p1 = V X x_1 = 43/13 X 6/43 = 6/13
p2 = V X x_2 = 43/13 X 0 = 0
p3 = V X x_3 = 43/13 X 7/43 = 7/13

Hence, company A's optimal strategy is (6/13, 0, 7/13).

Dominance

The principle of dominance states that if one strategy of a player dominates over the other strategy in all conditions then the later strategy can be ignored. A strategy dominates over the other only if it is preferable over other in all conditions. The concept of dominance is especially useful for the evaluation of two-person zero-sum games where a saddle point does not exist.

Generally, the dominance property is used to reduce the size of a large payoff matrix.

Rules

- If all the elements of a column (say i_{th} column) are greater than or equal to the corresponding elements of any other column (say j_{th} column), then the i_{th} column is dominated by the j_{th} column and can be deleted from the matrix.
- If all the elements of a row (say i_{th} row) are less than or equal to the corresponding elements of any other row (say j_{th} row), then the i_{th} row is dominated by the j_{th} row and can be deleted from the matrix.

Example

		Player B			
		I	**II**	**III**	**IV**
	I	3	5	4	2
Player A	**II**	5	6	2	4
	III	2	1	4	0
	IV	3	3	5	2

Use the concept of dominance to solve this problem.

Solution.

		Player B				
Player A		**I**	**II**	**III**	**IV**	**Minimum**
	I	3	5	4	2	2

	II	5	6	2	4	2
	III	2	1	4	0	0
	IV	3	3	5	2	2
Maximum		5	6	5	4	

There is no **saddle point** in this game.

Using dominance property

If a column is greater than another column (compare corresponding elements), then delete that column.
Here, I and II column are greater than the IV column. So, player B has no incentive in using his I and II course of action.

	Player B		
		III	**IV**
Player A	**I**	4	2
	II	2	4
	III	4	0
	IV	5	2

If a row is smaller than another row (compare corresponding elements), then delete that row.
Here, I and III row are smaller than IV row. So, player A has no incentive in using his I and III course of action.

	Player B		
		III	**IV**
Player A	**II**	2	4
	IV	5	2

Now you can use any one of the following to determine the value of game

- Algebraic Method

- Calculus Method
- Linear Programming Method

(Try it yourself)

2 x n Games

Games where one player has only two courses of action while the other has more than two, are called 2 X n or n X 2 games. If these games do not have a saddle point or are reducible by the dominance method, then before solving these games we write all 2 X 2 sub-games and determine the value of each 2 X 2 sub-game. This method is illustrated by the following example.


Example

Determine the solution of game for the pay-off matrix given below:

	Player B			
		I	**II**	**III**
Player A	**I**	-3	-1	7
	II	4	1	-2

Solution.

Obviously, there is no saddle point and also no course of action dominates the other. Therefore, we consider each 2 X 2 sub-game and obtain their values.

(a)

	Player B		
		I	**II**
Player A	**I**	-3	-1
	II	4	1

The saddle point is 1. So the value of game, V1 is 1.

(b)

	Player B		
Player A		I	II
	I	-3	7
	II	4	-2

This game has no saddle point, so we use the algebraic method.

$$\text{Value of game, V2} = \frac{(-3) \times (-2) - (7 \times 4)}{(-3 - 2) - (7 + 4)} = \frac{11}{8}$$

(c)

	Player B		
Player A		II	III
	I	-1	7
	II	1	-2

This game has no saddle point, so we use the algebraic method.

$$\text{Value of game, V3} = \frac{(-1) \times (-2) - (7 \times 1)}{(-1 - 2) - (7 + 1)} = \frac{5}{11}$$

The 2 X 2 sub-game with the lowest value is (c) and hence the solution to this game provides the solution to the larger game.

Using algebraic method:

A plays (3/11, 8/11)
B plays (0, 9/11, 2/11)
Value of game is 5/11.

Graphical Method

The method discussed in the previous section is feasible when the value of *n* is small, because the larger value of *n* will yield a larger number of 2 X 2 sub-games. In this section, we discuss another method for solving 2 X n games. This method can only be used in games with no saddle point, and having a pay-off matrix of type n X 2 or 2 X n.

Example

Consider the following pay-off matrix

		Player B	
		B_1	**B_2**
Player A	**A_1**	-2	4
	A_1	8	3
	A_1	9	0

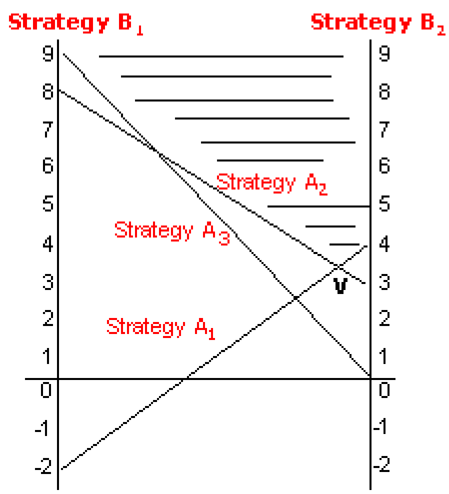
Solution.

The game does not have a saddle point as shown in the following table.

		Player B		Minimum	Probability
		B_1	**B_2**		
Player A	**A_1**	-2	4	-2	q_1
	A_2	8	3	3	q_2
	A_3	9	0	0	q_3
Maximum		9	4		
Probability		p_1	p_1		

Maximin = 4, Minimax = 3

First, we draw two parallel lines 1 unit distance apart and mark a scale on each. The two parallel lines represent strategies of player B.
If player A selects strategy A1, player B can win –2 (i.e., loose 2 units) or 4 units depending on B's selection of strategies. The value -2 is plotted along the vertical axis under strategy B_1 and the value 4 is plotted along the vertical axis under strategy B_2. A straight line joining the two points is then drawn.
Similarly, we can plot strategies A_2 and A_3 also. The problem is graphed in the following figure.



The lowest point V in the shaded region indicates the value of game. From the above figure, the value of the game is 3.4 units. Likewise, we can draw a graph for player B.

The point of optimal solution (i.e., maximin point) occurs at the intersection of two lines:

$E1 = -2p_1 + 4p_2$ and
$E2 = 8p_1 + 3p_2$

Comparing the above two equations, we have

$-2p_1 + 4p_2 = 8p_1 + 3p_2$

Substituting $p_2 = 1 - p_1$
$-2p_1 + 4(1 - p_1) = 8p_1 + 3(1 - p_1)$
$p_1 = 1/11$
$p_2 = 10/11$

Substituting the values of p_1 and p_2 in equation E1

V = -2 (1/11) + 4 (10/11) = 3.4 units

Bidding Problems

Several competitive situations involve bidding for contracts, tenders, licenses, etc. All the bidding problems may be classified into two groups:

- Auction Bids - In case of auction bids, the bids are open.
- Closed Bids - In case of closed bids, each bidder submits his bid in a closed envelope.

1. Auction Bids or Open Bids

Example

A chair and a table worth Rs. 80 and Rs.120 are to be auctioned at a public sale. There are only two bidders - Vinay and Manish. Vinay and Manish have Rs. 100 and Rs. 130 respectively. What should be their strategies, if each bidder is interested in maximizing his return ? Assume that both the bidders have complete information about each other's money position.

Solution.

Suppose the bid increases successively by the amount Rs. Δ. It should be noted that at any bid, each player has a option to increase the bid or to leave the opponent's bid stand.

Suppose Manish has bid of Rs. x on the chair.

Case I

If Vinay allows Manish to win the chair for Rs. x then Manish will have only (130 - x) rupees for bidding on the table. Thus, he can not make his bid for the table more than Rs. 130 - x. Hence, Vinay will definitely win the table in Rs. $(130 - x + \Delta)$. Thus, Vinay's gain when he allows Manish to win the chair for Rs. x is

Rs. $[120 - (130 - x + \Delta)]$
$= $ Rs. $(x - \Delta - 10)$

Case II

To the contrary, if Vinay bids Rs. $x + \Delta$ for the chair and Manish allows him to win at this bid, then Vinay's gain is
Rs. $[80 - (x + \Delta)]$
$= 80 - x - \Delta$

Now since Vinay wants to maximize his return, he should bid $x + \Delta$ for the chair provided
$80 - x - \Delta \geq x - \Delta - 10$

= 2x ≤ 90
= x ≤ 45

Thus, till x ≤ 45, Vinay should bid for chair. When x > 45, he should allow Manish to win the chair for that bid.

Likewise, in the two cases, Manish's gains are
[120 - (100 -y) - Δ] and [80 - (y + Δ)]
Where y is the Vinay's bid for the chair

Thus, Manish should bid y + Δ for the chair provided

80 - (y + Δ) ≥ 120 - (100 -y) - Δ
= y ≤ 30

Obviously, Vinay will take the chair in Rs. 30 because he can increase his bid without any loss upto Rs. 45, Manish will take the table in Rs. (100 - 30) = 70 because Vinay, after winning the chair for Rs. 30 cannot increase his bid for the table more than Rs. 70. Thus, Manish will get the table for Rs. 70. The gain of Vinay is Rs. (80 - 30) = Rs. 50, and of Manish is Rs. (120 - 70) = Rs. 50.

2. Closed Bids


Example

A joystick and a keyboard worth Rs 80. and Rs. 100 are to be bid simultaneously by two bidders A and B. Both have intention of devoting a total sum of Rs. 110 for the items. If each uses a minimax criterion, find the resulting bids.

Solution.

Since the bids are to be made simultaneously, they are closed bids.

Suppose P and Q are A's best bids for the joystick and the keyboard respectively. A's best bids are those which give the same profit to A on both the items. If t is the total profit associated with a successful bid, then

2t = (80 - P) + (100 - Q)
2t = 180 -(P + Q)
2t = 180 -110
2t = 70
t = 35

P = 80 - t
= 80 - 35
= 45
and

Q =100 - t
= 100 - 35
= 65.

Thus, optimal bids for A are Rs. 45 for joystick, and Rs. 65 for keyboard.
The optimal bids for B will be same as A's optimal bids.

Advantages & Limitations of Game Theory

Advantages

- Game theory gives insight into several less-known aspects, which arise in situations of conflicting interests. For example, it describes and explains the phenomena of bargaining and coalition-formation.
- Game theory develops a framework for analyzing decision making in such situations where interdependence of firms is considered.
- At least in two-person zero-sum games, game theory outlines a scientific quantitative technique that can be used by players to arrive at an optimal strategy.

Limitations

- The assumption that players have the knowledge about their own pay-offs and pay-offs of others is not practical.
- The techniques of solving games involving mixed strategies particularly in case of large pay-off matrix is very complicated.
- All the competitive problems cannot be analyzed with the help of game theory.